Modeling Arctic Sea Ice

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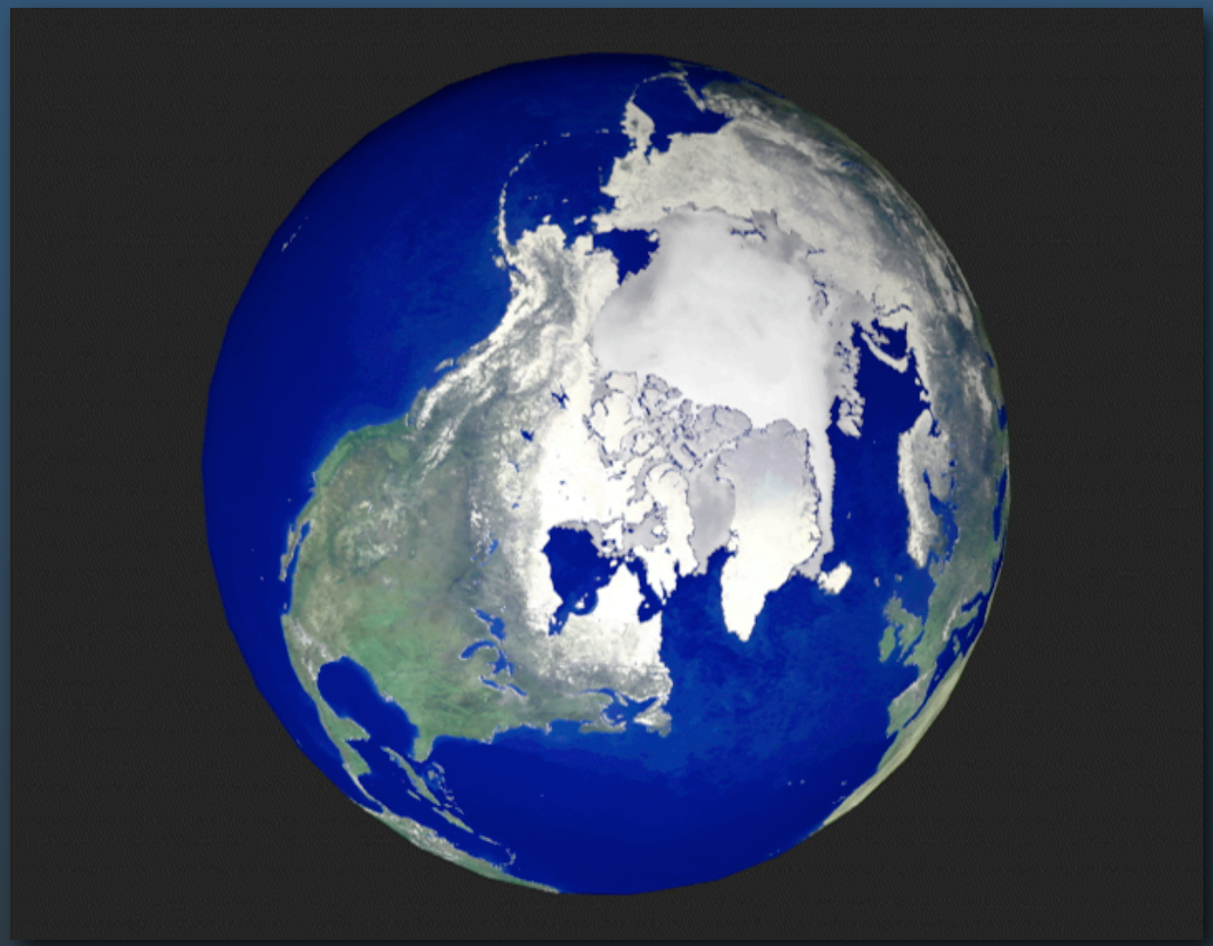
Outline



- Introduction/Motivation
- Material-Point Method
- Constitutive Model
- Kinematics
- Numerical Simulations
- Conclusions

Why Model Sea Ice?

- Forcasting: shipping, safety and environmental remediation.

  Where is the ice?

  How fast is it moving?

  Where is it going?

- Climatology: global climate models

  How thick is the ice?

  What is its extent?

  How much is new?

- Ice deformation provides the mechanical input to the ice thickness distribution.

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Why a new ice model?

The viscous-plastic model is an isotropic model based on a 100 km scale in which it was assumed that cracks, ridges and leads were randomly distributed.

- Satellite images show large ice deformation events occurring in long-lasting linear features that appear to correspond to displacement (or velocity) discontinuities in the deformation field due to leads
- Most 10 km Lagrangian cells do not have permanent deformation during the year

(R. Kwok, J. Geophys. Res., Vol. 111, No. C11, C11S22, 2006)

For thick first-year ice and multi-year ice, we assume most deformation occurs due to discontinuities in the displacement field.

Equations of Motion

$$(\rho h)\frac{d\mathbf{v}}{dt} - \mathbf{t}_a - \mathbf{t}_w + (\rho h) f_c(\mathbf{e}_3 \times \mathbf{v}) - \nabla \cdot (\boldsymbol{\sigma} h) = 0$$

Inertia Coriolis Stress div

Air Drag:

$$\mathbf{t}_a = c_a \rho_a \|\mathbf{v}_a\| \mathbf{R}_a \mathbf{v}_a \qquad \mathbf{R}_a = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

Water Drag:

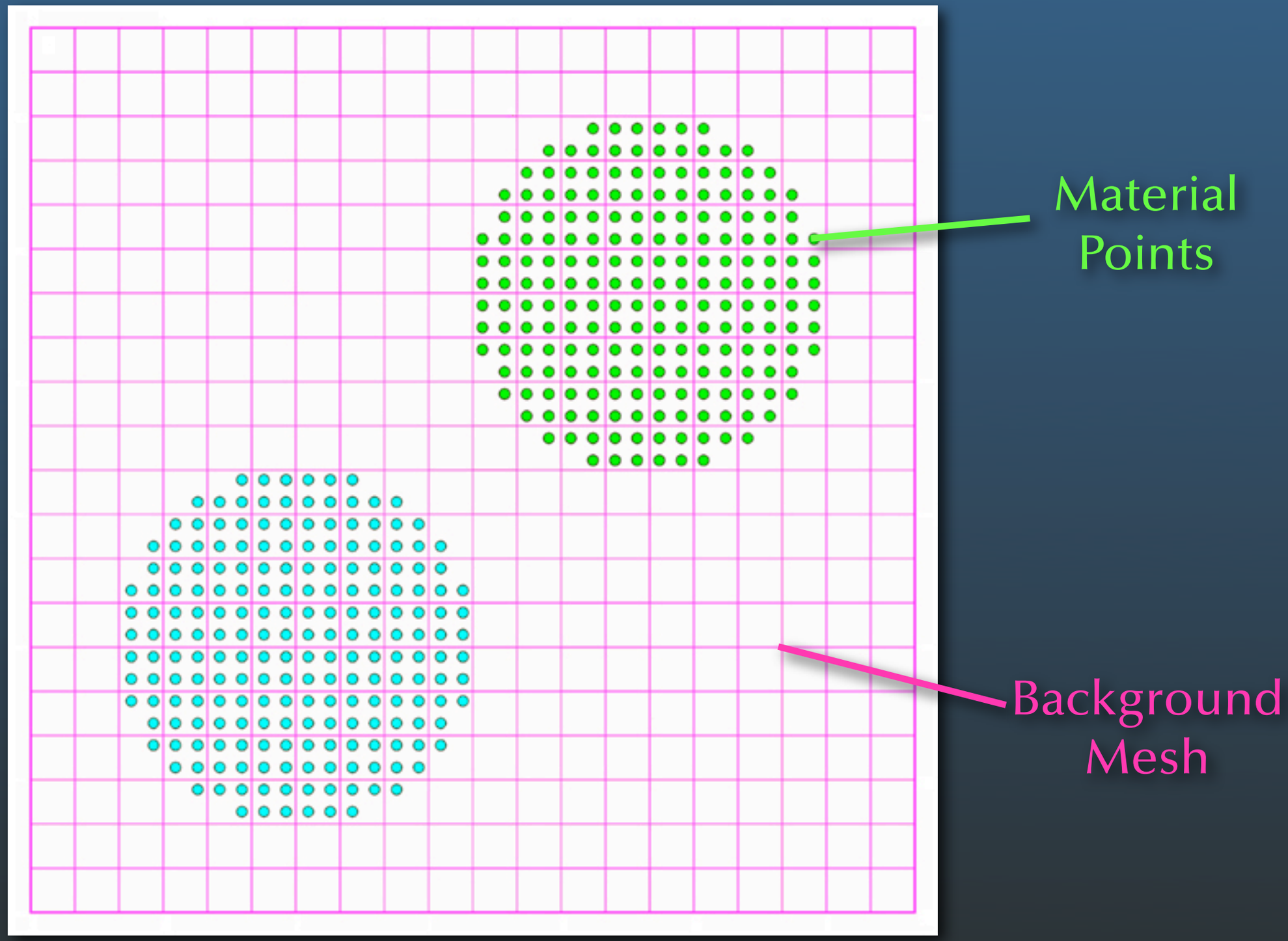
$$\mathbf{t}_w = c_w \rho_w \|\mathbf{v} - \mathbf{v}_w\| \mathbf{R}_w (\mathbf{v} - \mathbf{v}_w) \qquad \mathbf{R}_w = \begin{bmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{bmatrix}$$

Simulation Parameters

symbol	name	value
ρ	sea ice density (mass/vol)	917.0
h	sea ice thickness	3.0
f_c	coriolis parameter	1.46×10^{-4}
c_a	air drag coefficient	0.0012
ρ_a	air density	1.20
$\mathbf{v}_a$	wind velocity	from data
α	air turning angle	0.50
c_w	sea water drag coefficient	0.00536
ρ_w	sea water density	1026.0
β	sea water turning angle	0.0
$\mathbf{v}_w$	sea water velocity	from data
G	(linearized) elastic shear modulus	3.6765×10^5
K	(linearized) elastic bulk modulus	11.905×10^5
$\mathbf{e}_3$	out of plane vector	$(0,0,1)^T$
$\boldsymbol{\sigma}$	ice stress	– – –
$\mathbf{v}$	ice velocity	– – –

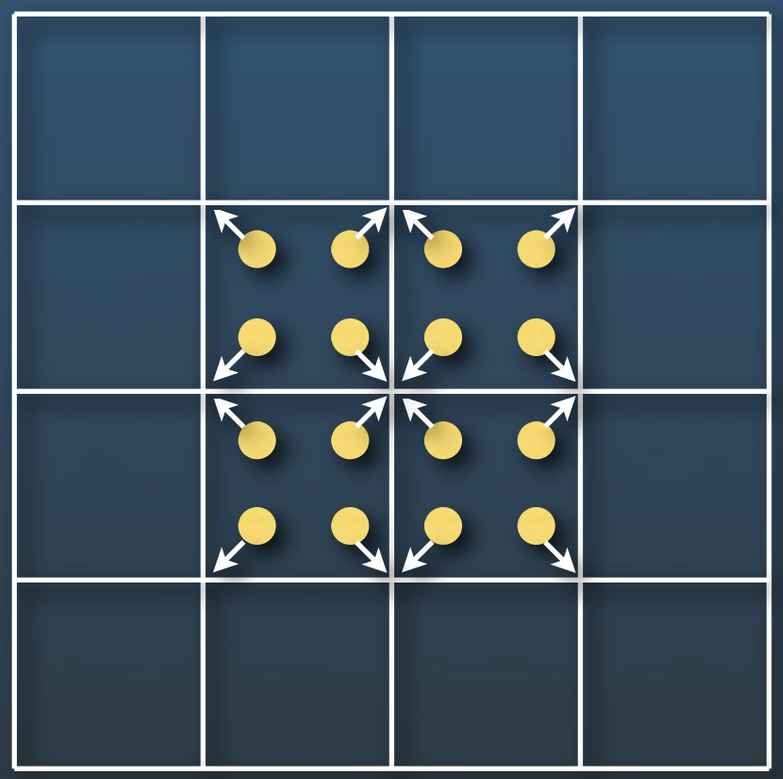
Units: m, kg, sec

Elements of an MPM Simulation



MPM Computational Cycle

Step 1: Interpolate material-point data to the mesh



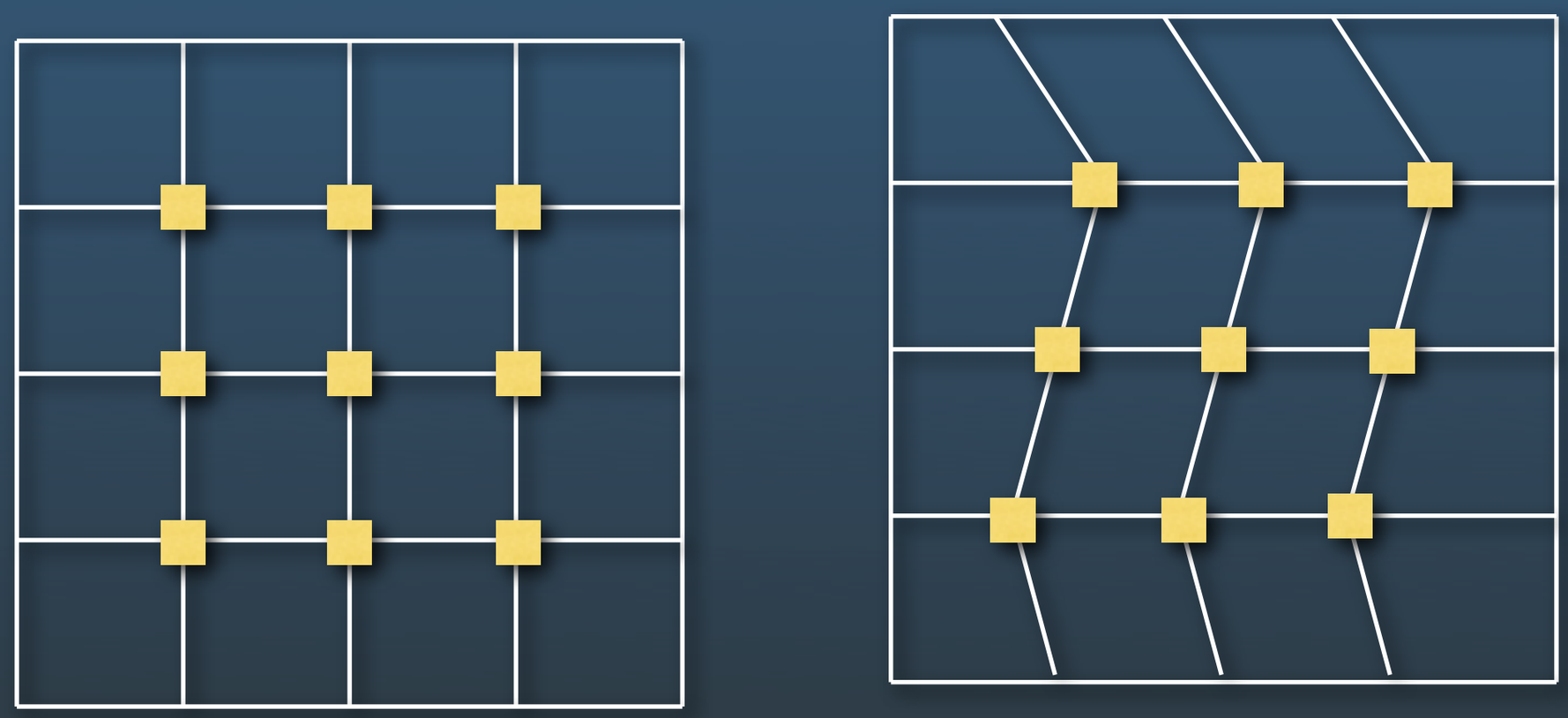
$$m_i = \sum_p m_p N_i(\mathbf{x}_p)$$

$$m_i \mathbf{v}_i = \sum_p m_p \mathbf{v}_p N_i(\mathbf{x}_p)$$

$$\mathbf{f}_i^{\text{int}} = -\sum_p \mathbf{G}_{pi}^T \boldsymbol{\sigma}_p m_p / \rho_p$$

MPM Computational Cycle

Step 2: Solve equations of motion in an updated Lagrangian frame on the mesh

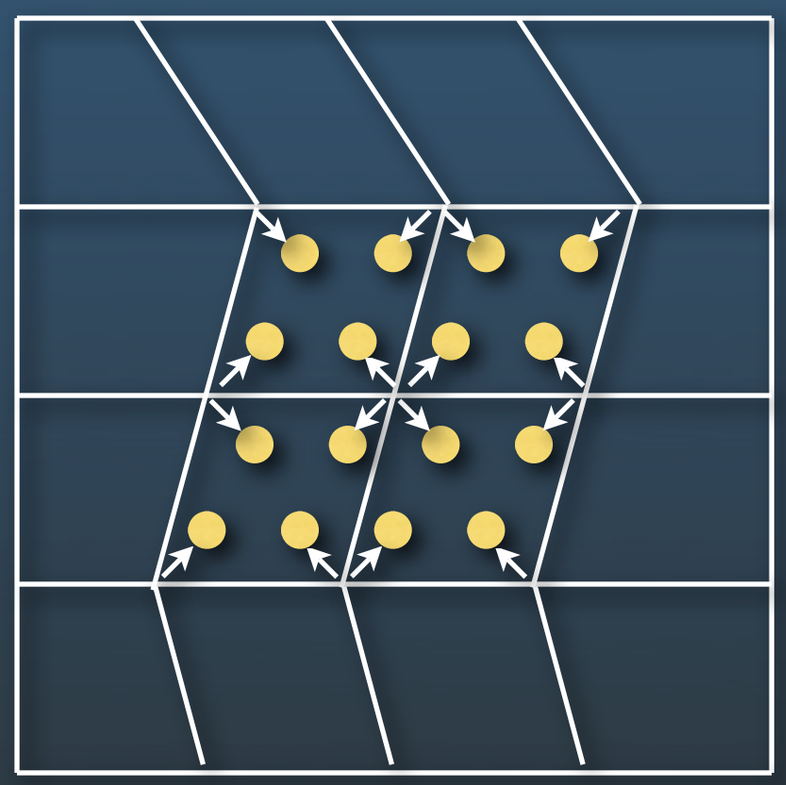


$$m_i \mathbf{a}_i = \mathbf{f}_i$$

$$\mathbf{v}_i \leftarrow \mathbf{v}_i + \Delta t \mathbf{a}_i$$

MPM Computational Cycle

Step 3: Update the material-points based on the mesh solution



$$\mathbf{x}_p \leftarrow \mathbf{x}_p + \Delta t \sum_i \mathbf{v}_i N_i(\mathbf{x}_p)$$

$$\mathbf{v}_p \leftarrow \mathbf{v}_p + \Delta t \sum_i \mathbf{a}_i N_i(\mathbf{x}_p)$$

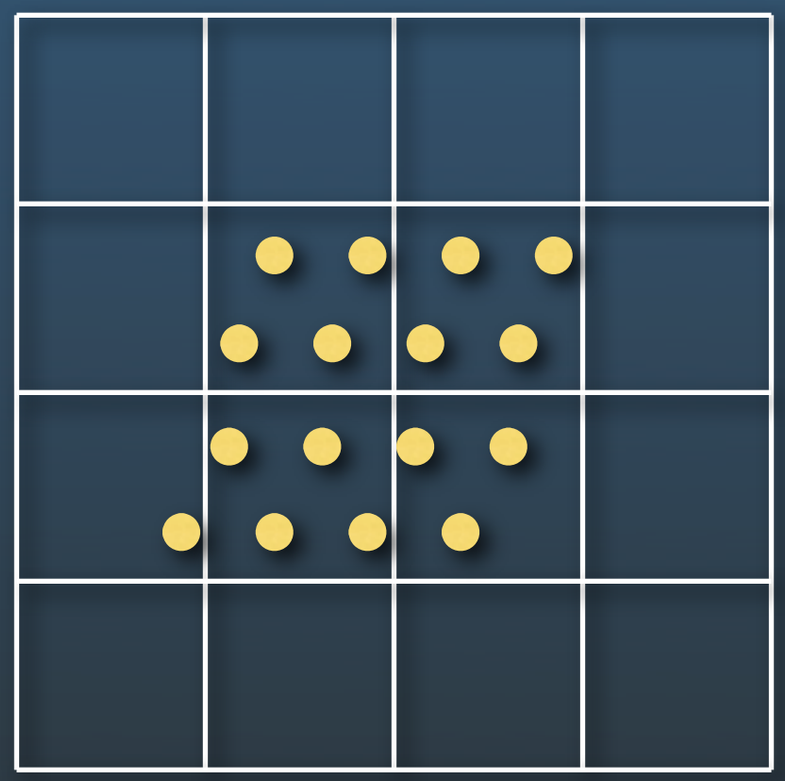
$$\mathbf{F}_p \leftarrow \mathbf{f}_p \mathbf{F}_p,$$

$$\mathbf{f}_p = \mathbf{I} + \Delta t \sum_i \mathbf{v}_i \nabla N_i(\mathbf{x}_p)$$

$$\boldsymbol{\sigma}_p = \ldots$$

MPM Computational Cycle

Step 4: Redefine the background mesh if desired



Features of MPM

Dual description of the continuum: material points and background computational mesh

The convective phase of the algorithm is performed by Lagrangian material points which carry position, mass, velocity…

The interaction between material points is solved using a finite element or finite difference discretization on a mesh (cost is linear in the number of material points)

Information is transferred between the material points and the mesh by interpolation (only changes are interpolated, keeping numerical dissipation relatively small)

Material points move in a continuous velocity field providing a natural no-slip contact algorithm

Can use any constitutive model

Rotation Test

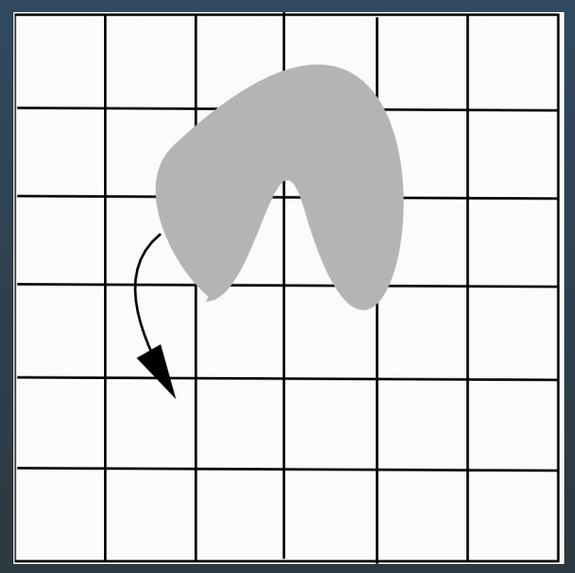
$$\frac{\partial h}{\partial t} + \mathbf{v} \cdot \nabla h = 0$$

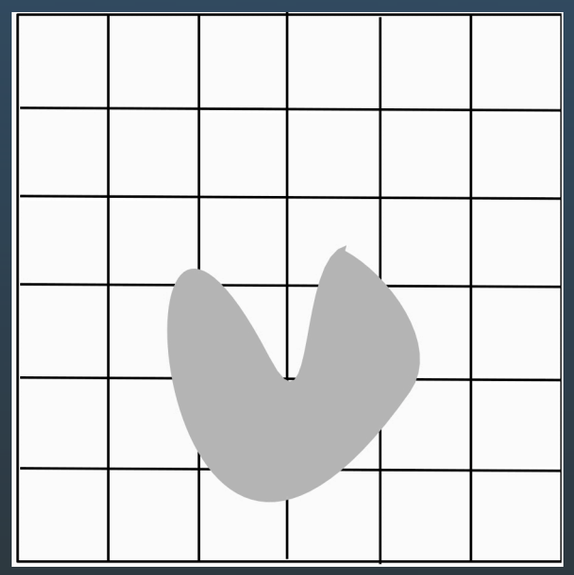
$$h(x, y, 0) = h_0(x, y)$$

or

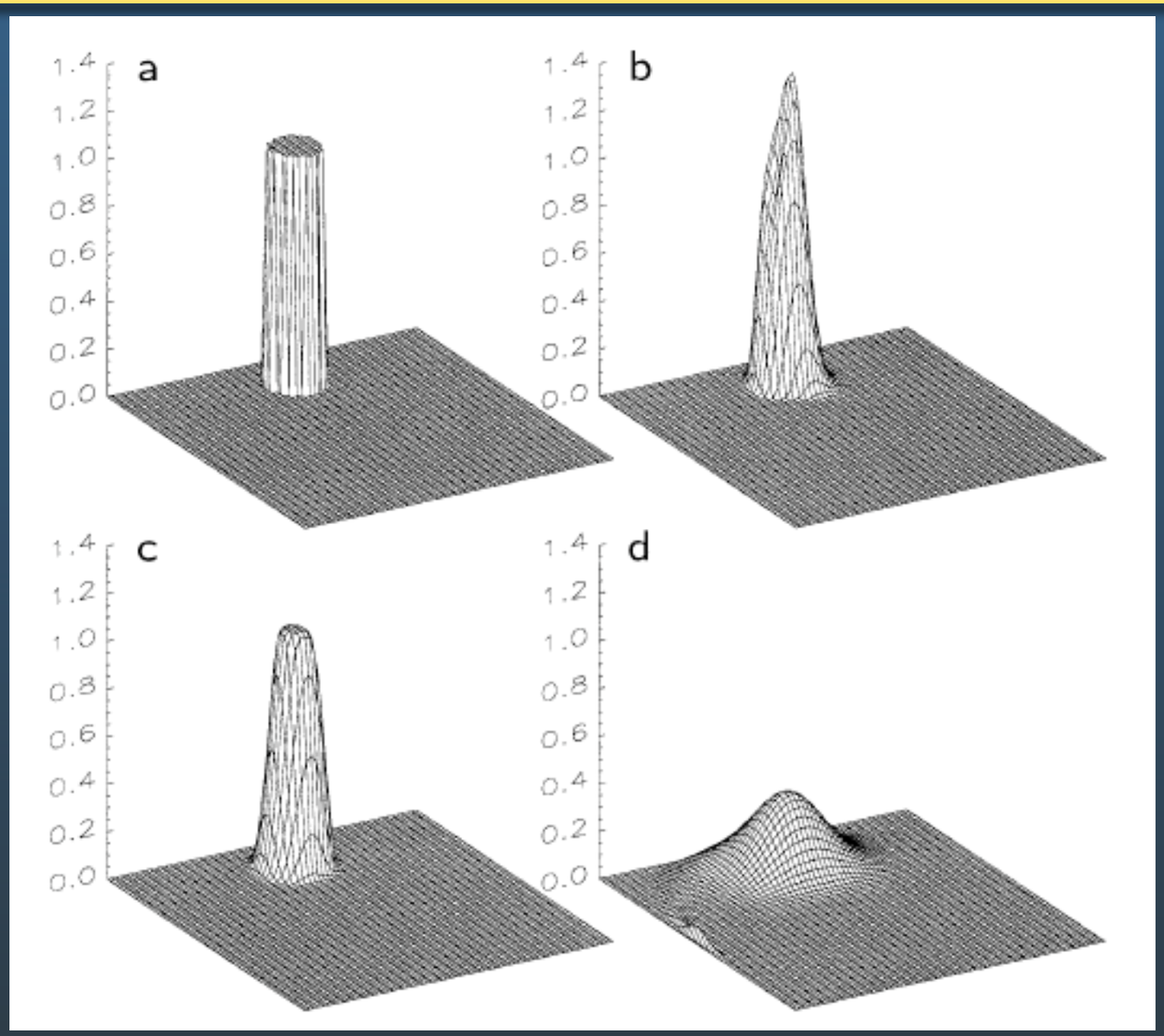
$$\frac{dh}{dt} = 0$$

$$h(x, y, 0) = h_0(x, y)$$





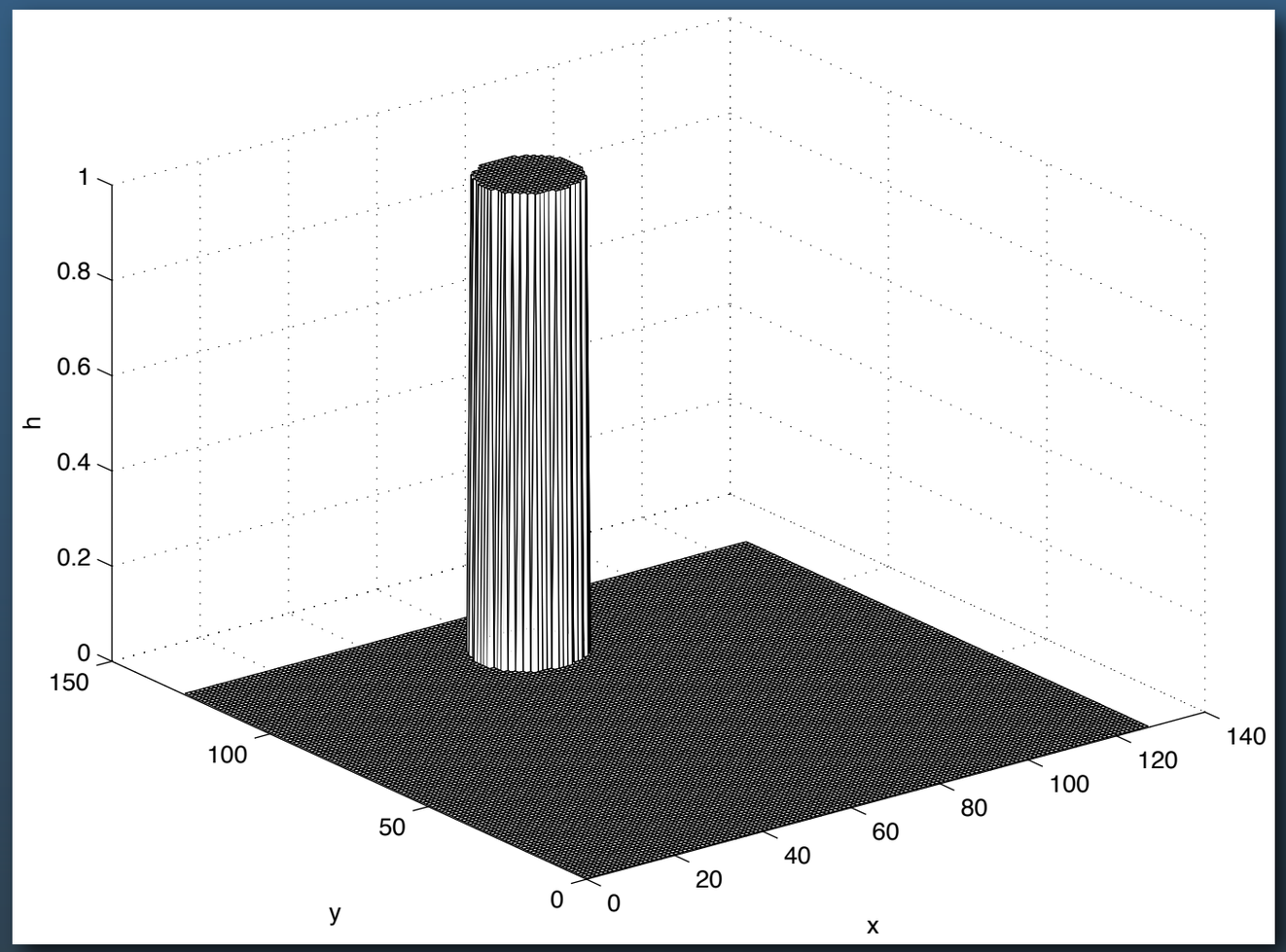
Finite Difference Results



(a) exact (b) MPDATA (c) remapping (d) upwind

Modeling Sea Ice Transport Using Incremental Remapping:
Lipscomb & Hunke (2004) Monthly Weather Review **132**:1341-1354

MPM Rotation Test



Final Configuration

Sulsky, D., H. Schreyer, K. Peterson, R. Kwok, M. Coon (2007), Using the Material-Point-Method to Model Sea Ice Dynamics, J. of Geophys. Res., 112, C02S90, doi: 10.1029/2005JC003329.

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Convergent Flow Test

$$\frac{\partial a}{\partial t} + \nabla \cdot (a\mathbf{v}) = 0$$

$$\frac{\partial v}{\partial t} + \nabla \cdot (v\mathbf{v}) = 0$$

$$\longrightarrow \quad \frac{\partial h}{\partial t} + \mathbf{v} \cdot \nabla h = 0$$

$$h = v / a$$

Let $\mathbf{v}(x, y) = -(x, 0)$

Exact Solution:

$$a(x, t) = e^t a_0(xe^t)$$

$$v(x, t) = e^t v_0(xe^t)$$

$$h(x, t) = h_0(xe^t)$$

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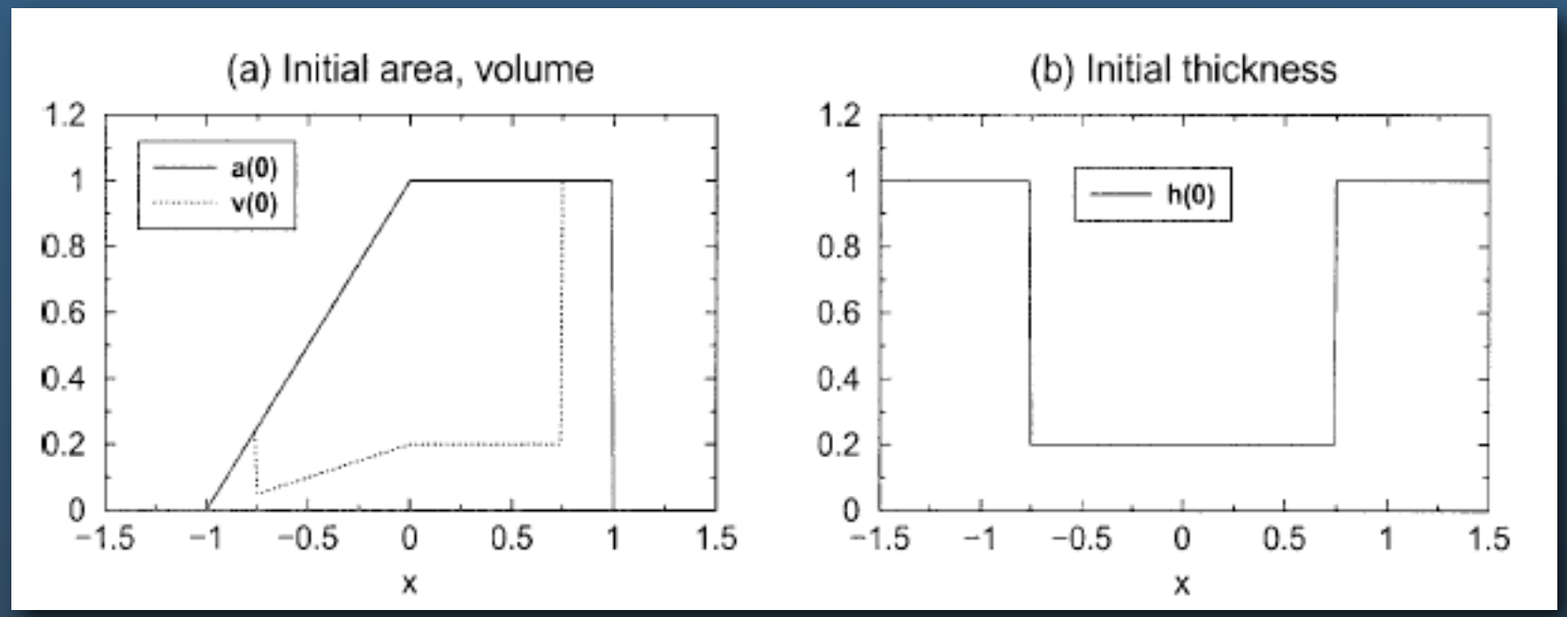
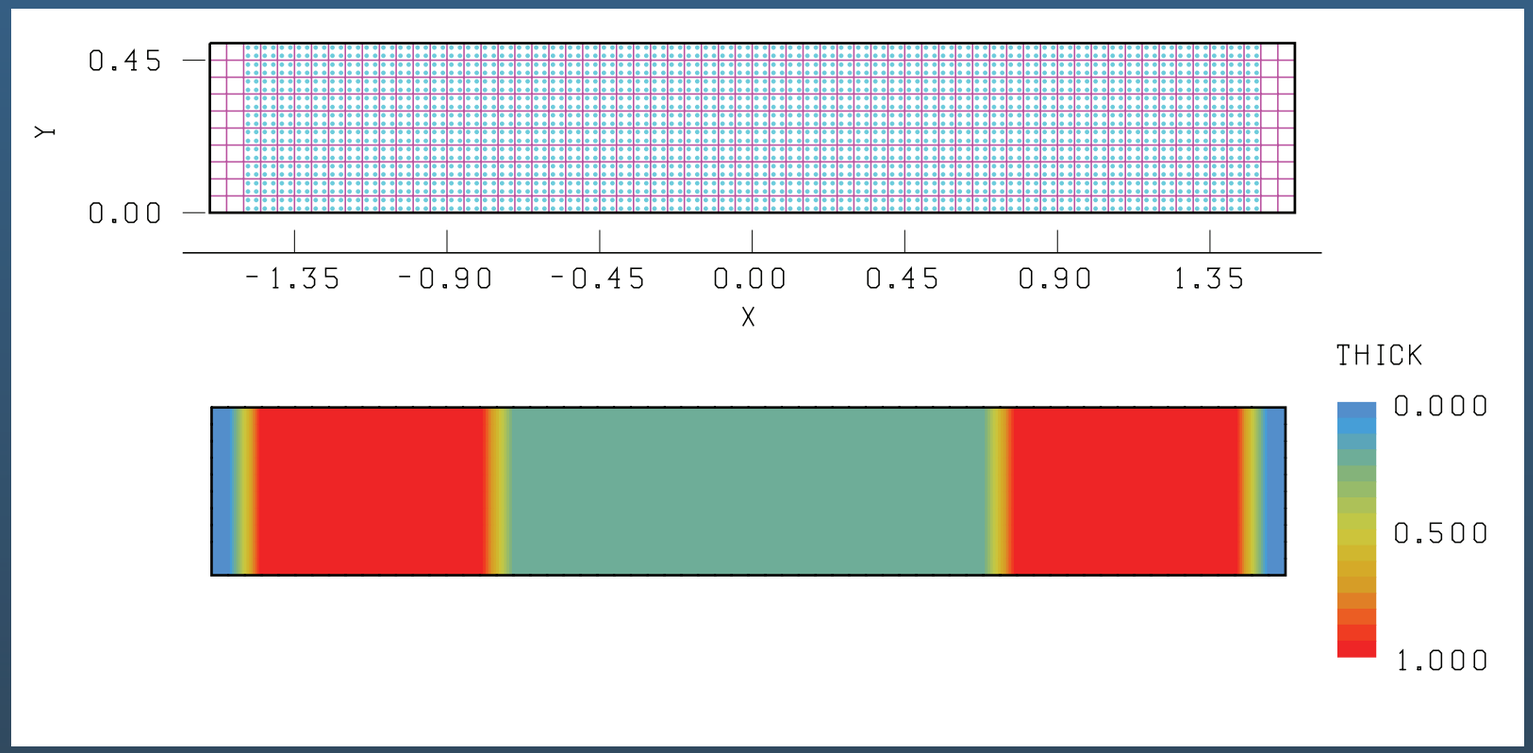
Convergent Flow Test

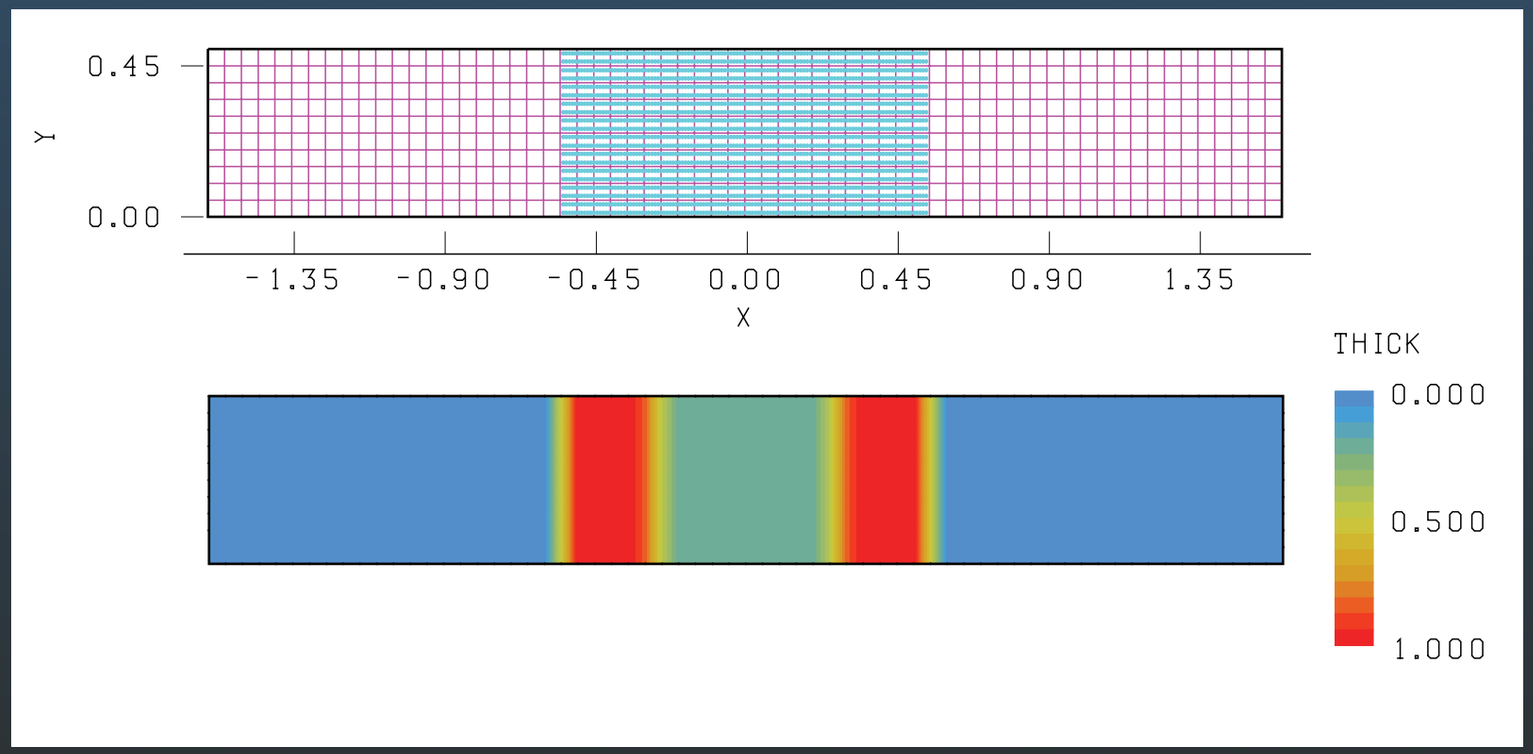


$$a_0(x) = \begin{cases} 1 + x & \text{for } -1 \le x \le 0 \\ 1 & \text{for } 0 \le x \le 1 \\ 0 & \text{otherwise} \end{cases} \qquad h_0(x) = \begin{cases} 1 & \text{for } |x| \ge 0.75 \\ 0.2 & \text{otherwise} \end{cases}$$

Modeling Sea Ice Transport Using Incremental Remapping:
Lipscomb & Hunke (2004) Monthly Weather Review **132**:1341-1354

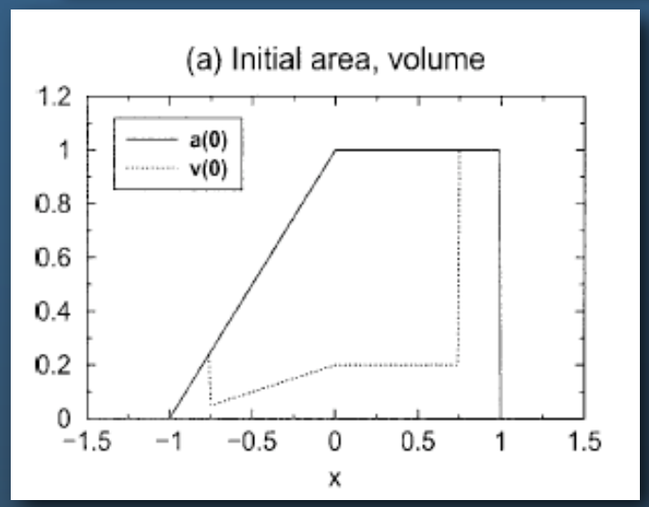
Convergent Flow Test

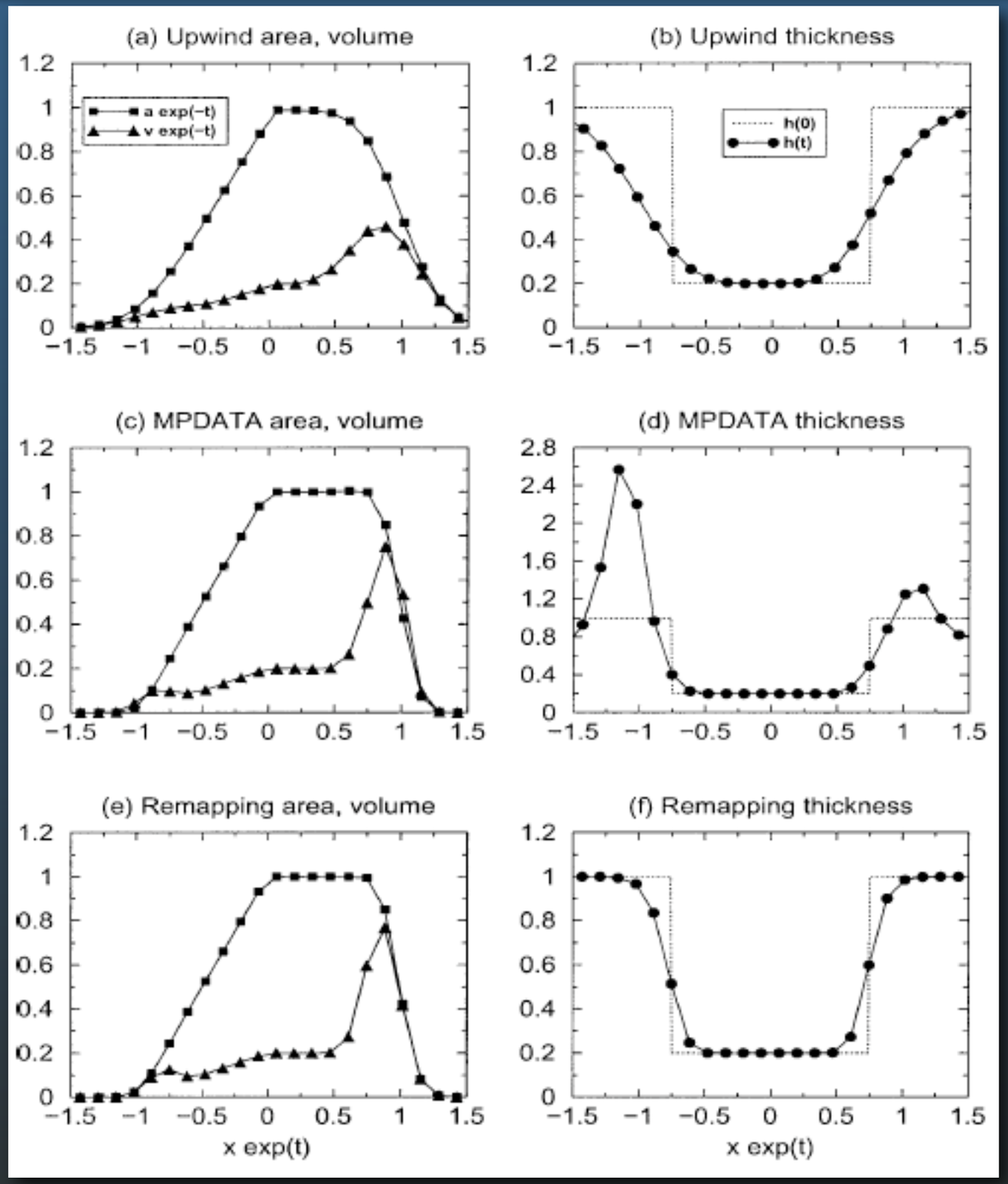




$\Delta x=0.05$

Finite Difference Results





Δx=0.05

Modeling Sea Ice Transport Using Incremental Remapping: Lipscomb & Hunke (2004) Monthly Weather Review **132**:1341-1354

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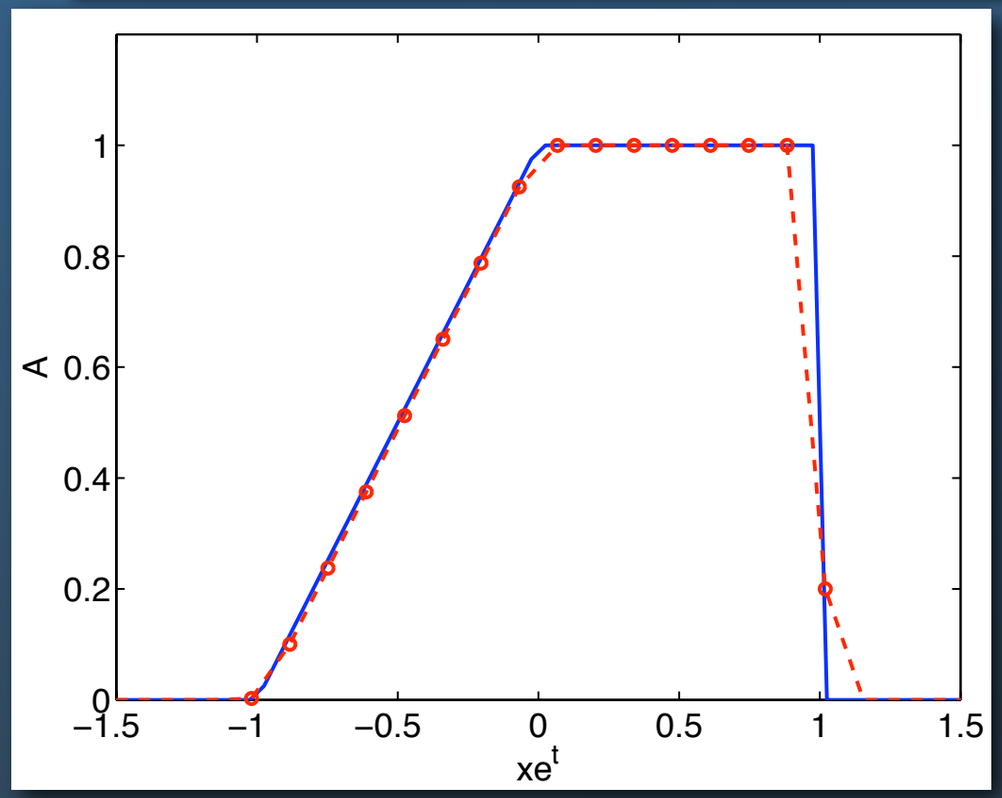
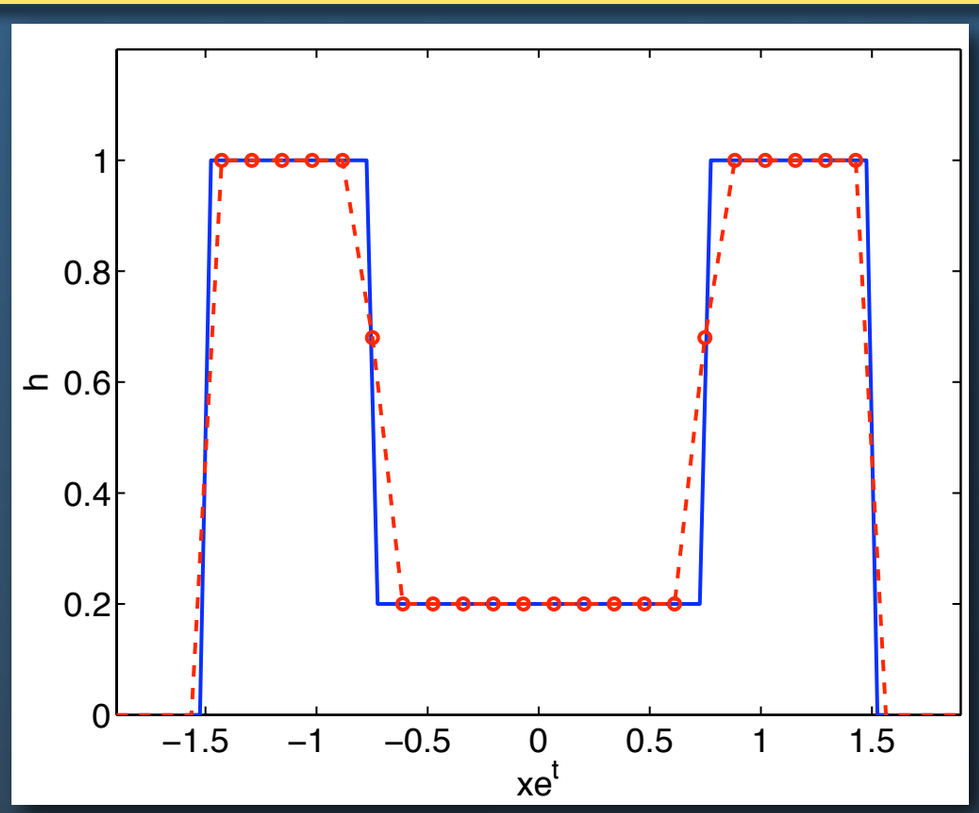
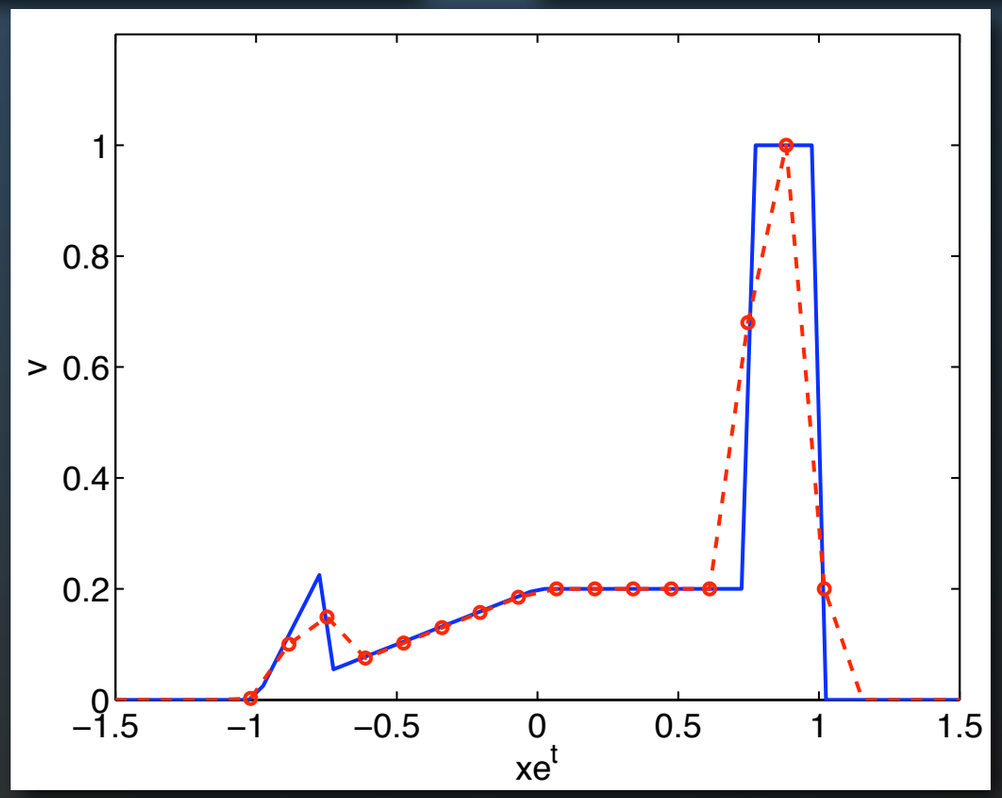
MPM Results $\Delta x=0.05$







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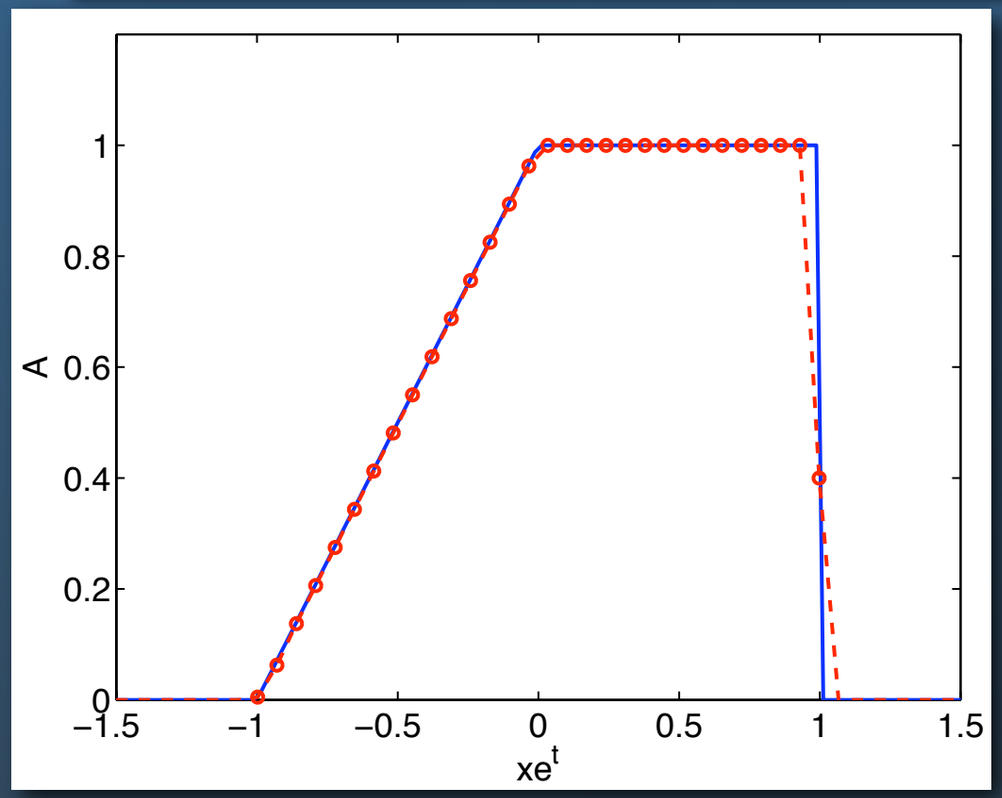
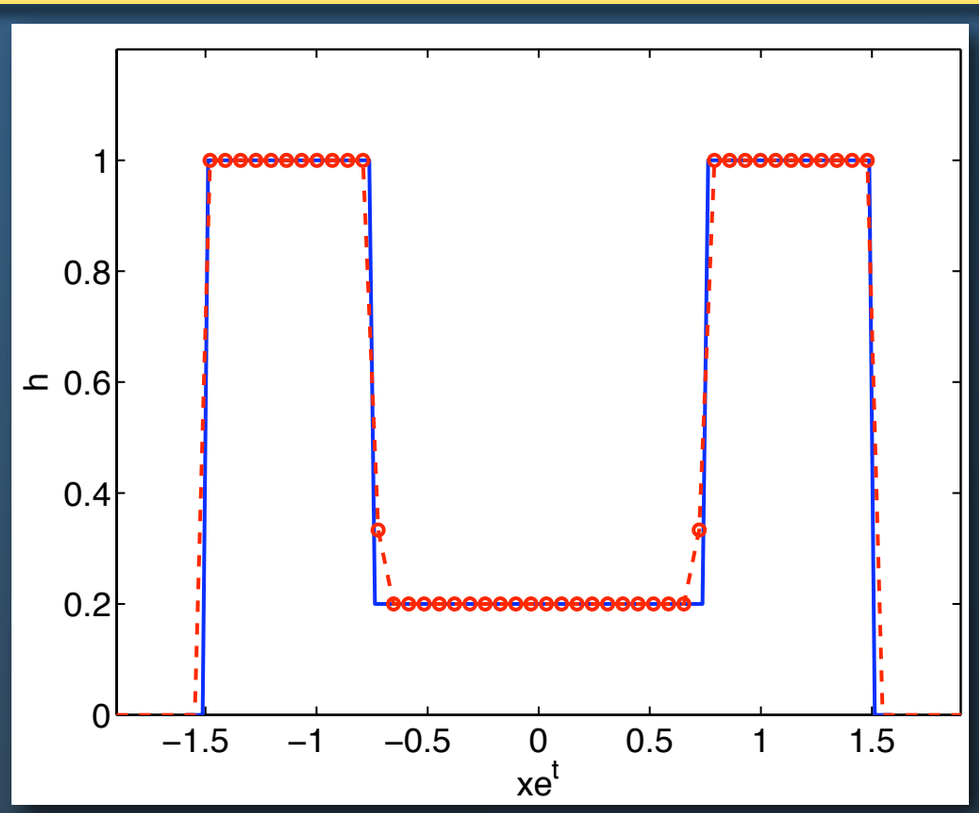
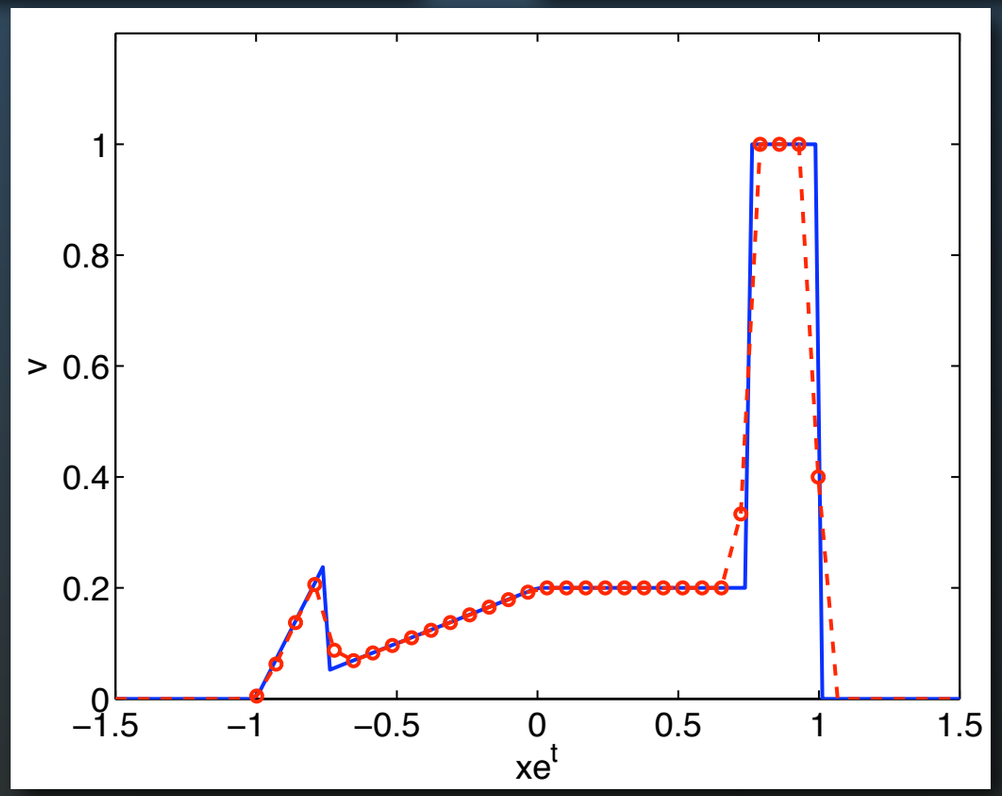
MPM Results $\Delta x=0.025$







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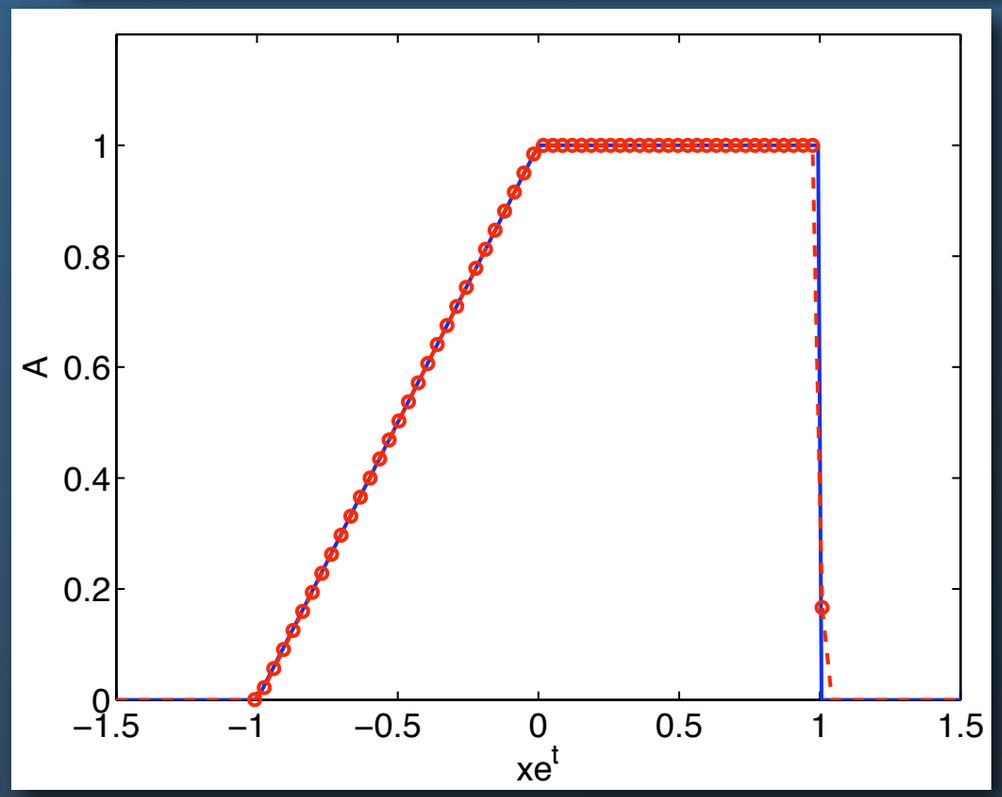
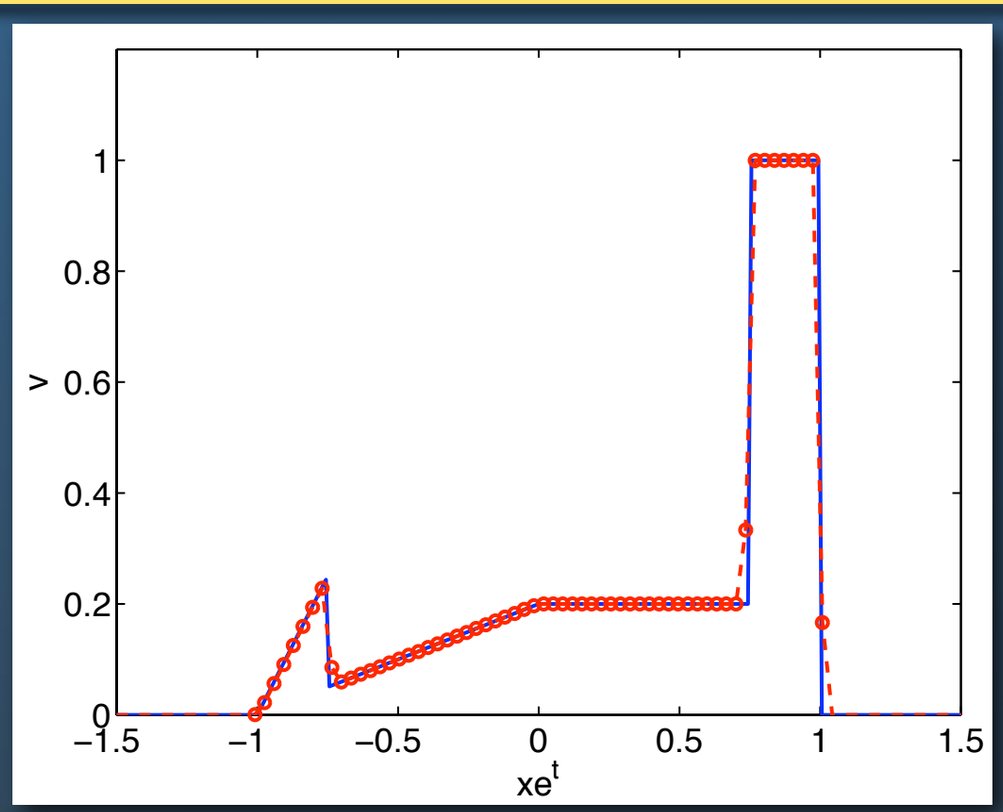
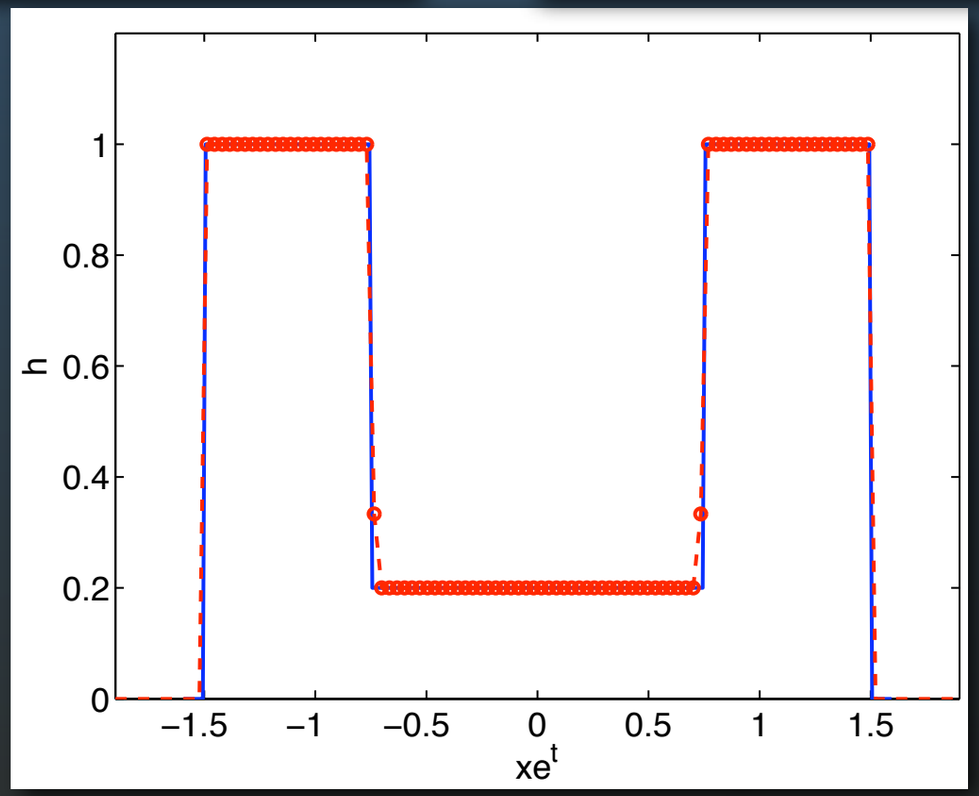
MPM Results Δx=0.0125







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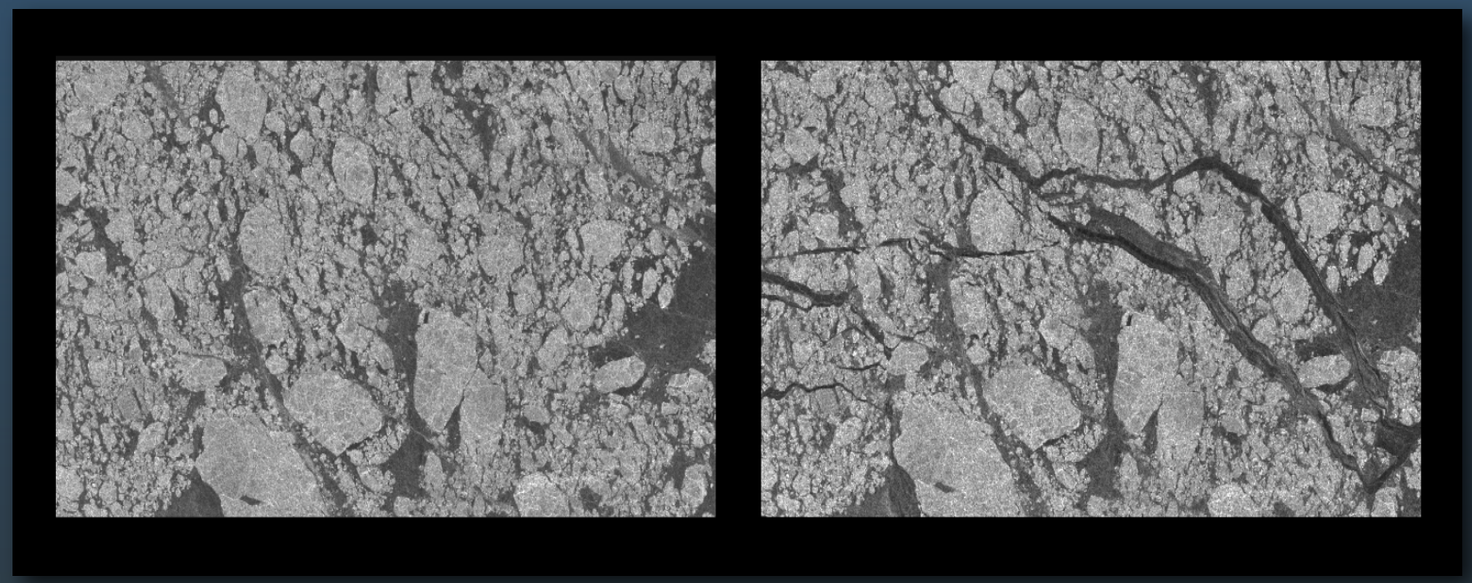
Goal

A numerically efficient sea ice model that includes observed features such as leads and ridges and uses available satellite data for verification

Lead

Ridge



http:www.jpl.nasa.gov/pictures/seaice

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Constitutive Model for Sea Ice

Initial Focus: Prediction and appearance of leads

Proposed Approach: Elastic-Decohesive Model

- Model intact ice as an elastic material
- Model leads as a discontinuity (decohesion)

(Ice is quasibrittle so we can borrow from models of concrete and rock)

Elastic-Decohesive Model

Similar to elastic-plastic model:

- damage surface ~ yield surface
- damage surface gives stress state at which a lead begins
- damage surface gives orientation of lead
- damage surface constructed from empirical data

Capture essential properties:

- correct energy dissipation
- correct peak stress
- keep method numerical tractable

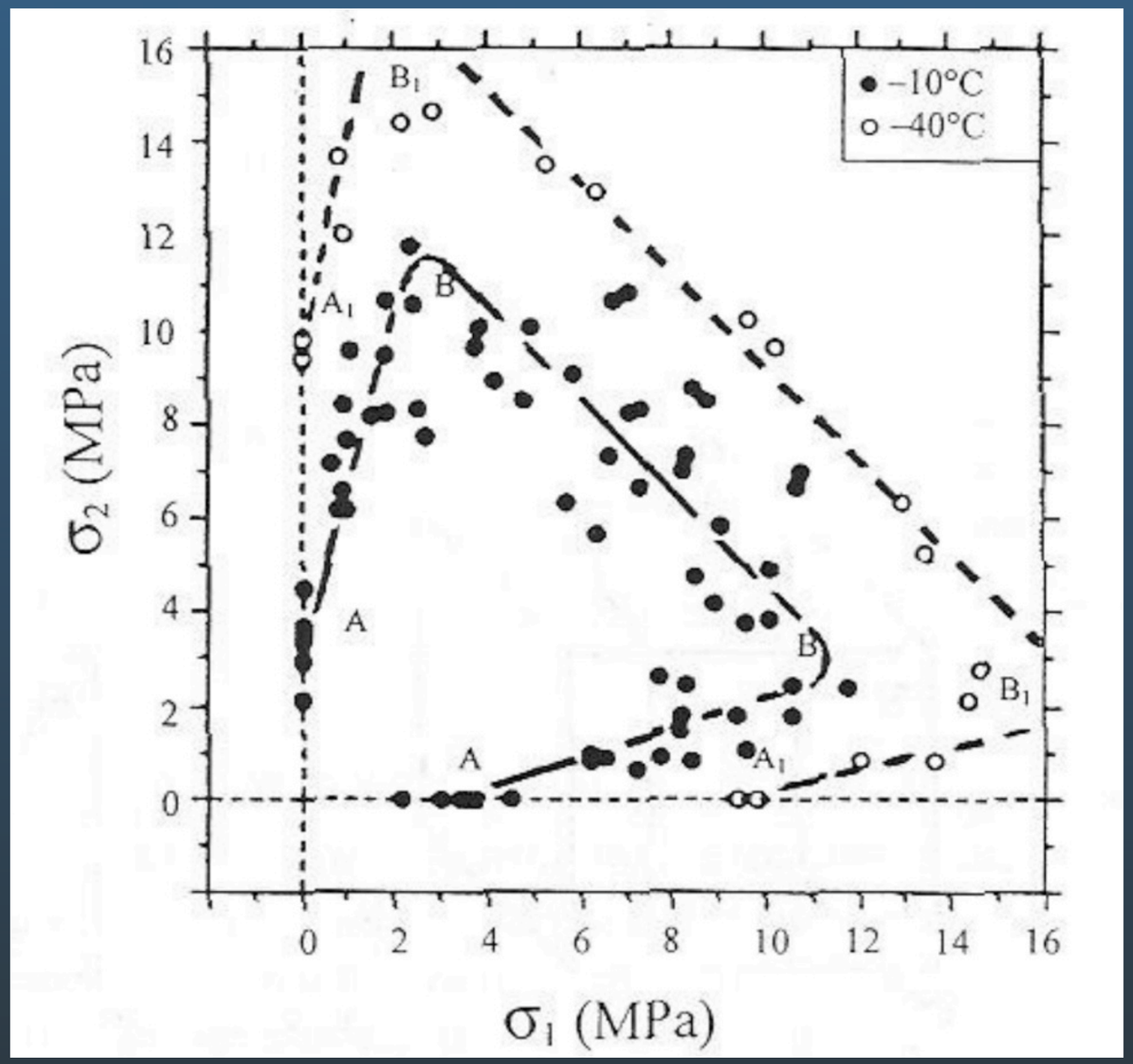
Elastic-Decohesive Model

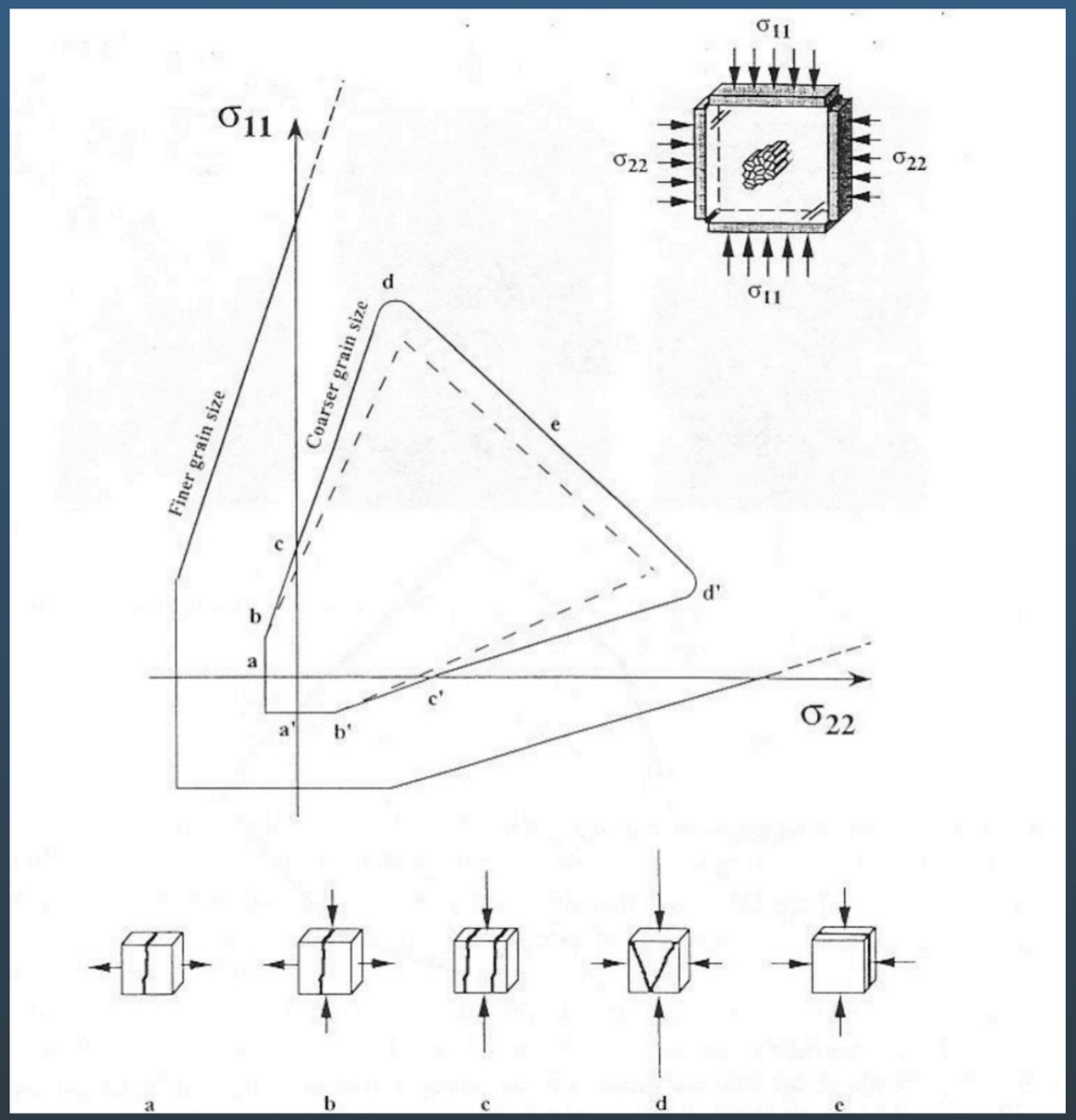
Components:

- Initiate Failure
- Determine lead orientation
- Evolve failure (create an open lead)

Failure surface informed by empirical data (Schulson) and in situ data (Coon)

Laboratory Data





Schulson, E. M. (2001) Brittle failure of ice, Engng. Fract Mech., **68**:1879-1887

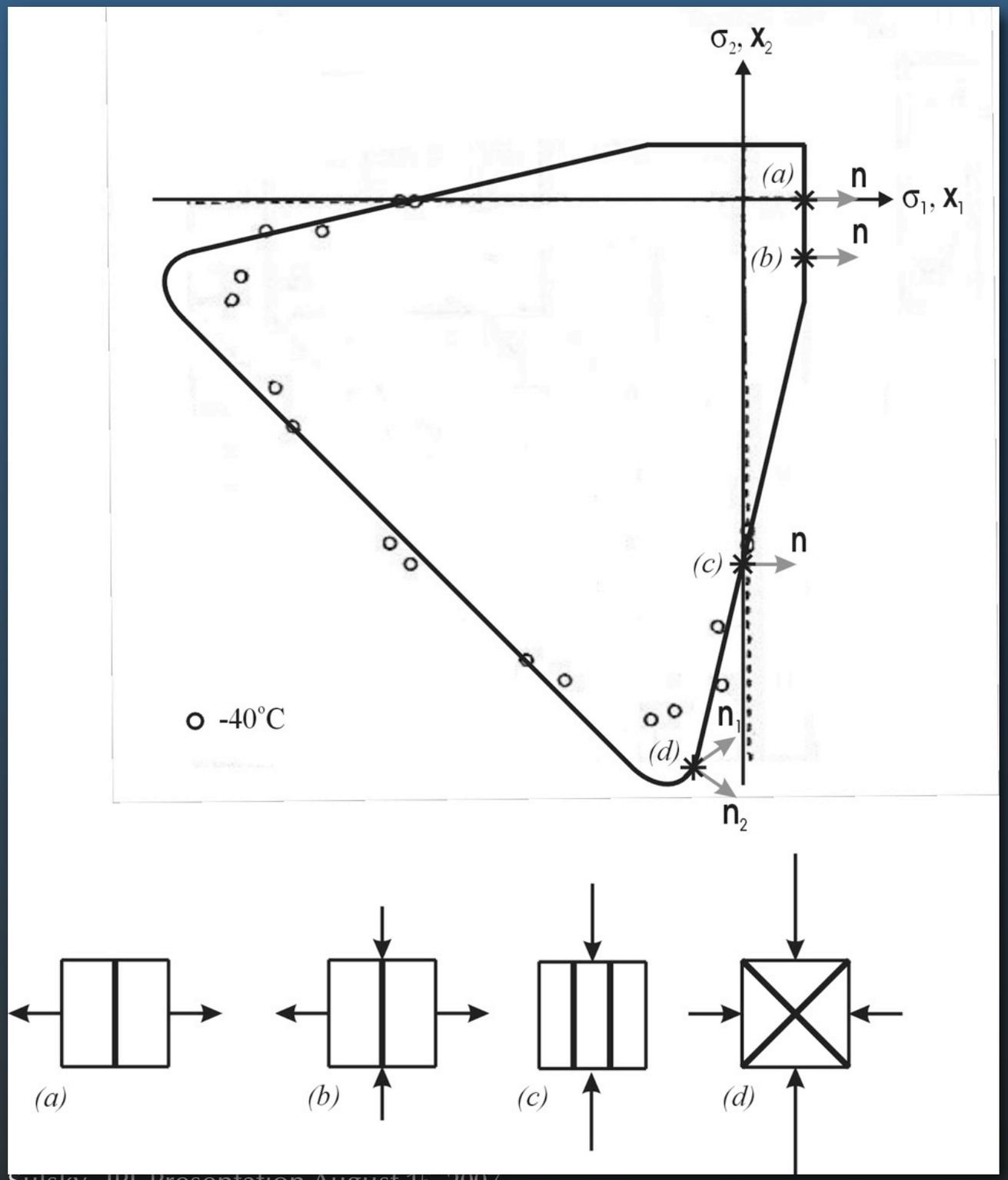
Stress at Failure - Failure Initiation

The failure envelope in stress space that describes initiation of failure is

$$F(\boldsymbol{\sigma}) = 0$$

What is *F* ?

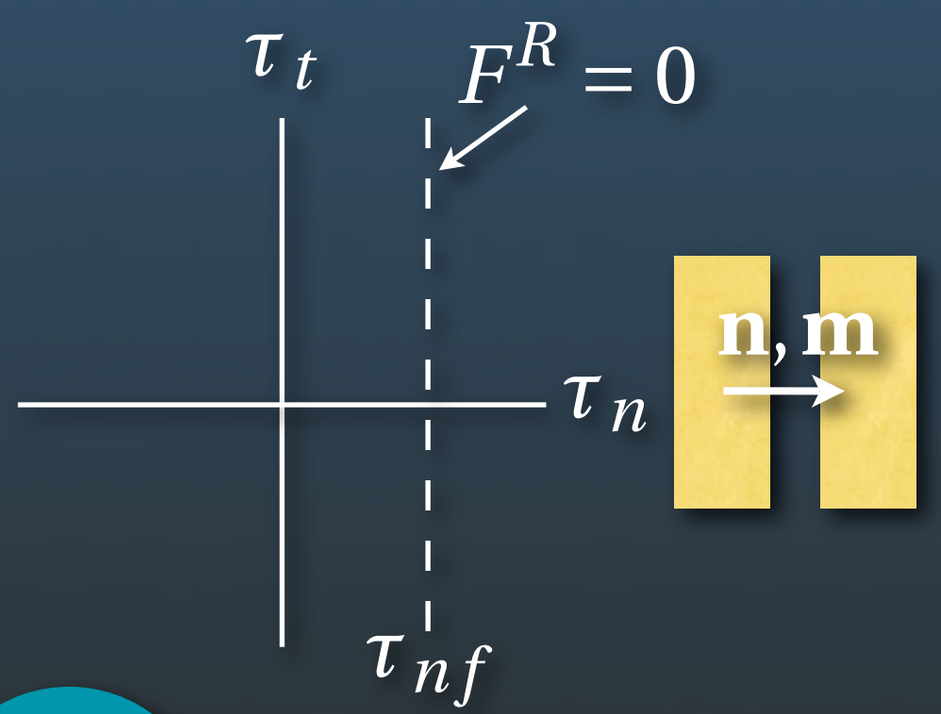
Schulson, E. Brittle Failure of Ice. Engineering Fracture Mechanics, 68:1839-1887, 2001.

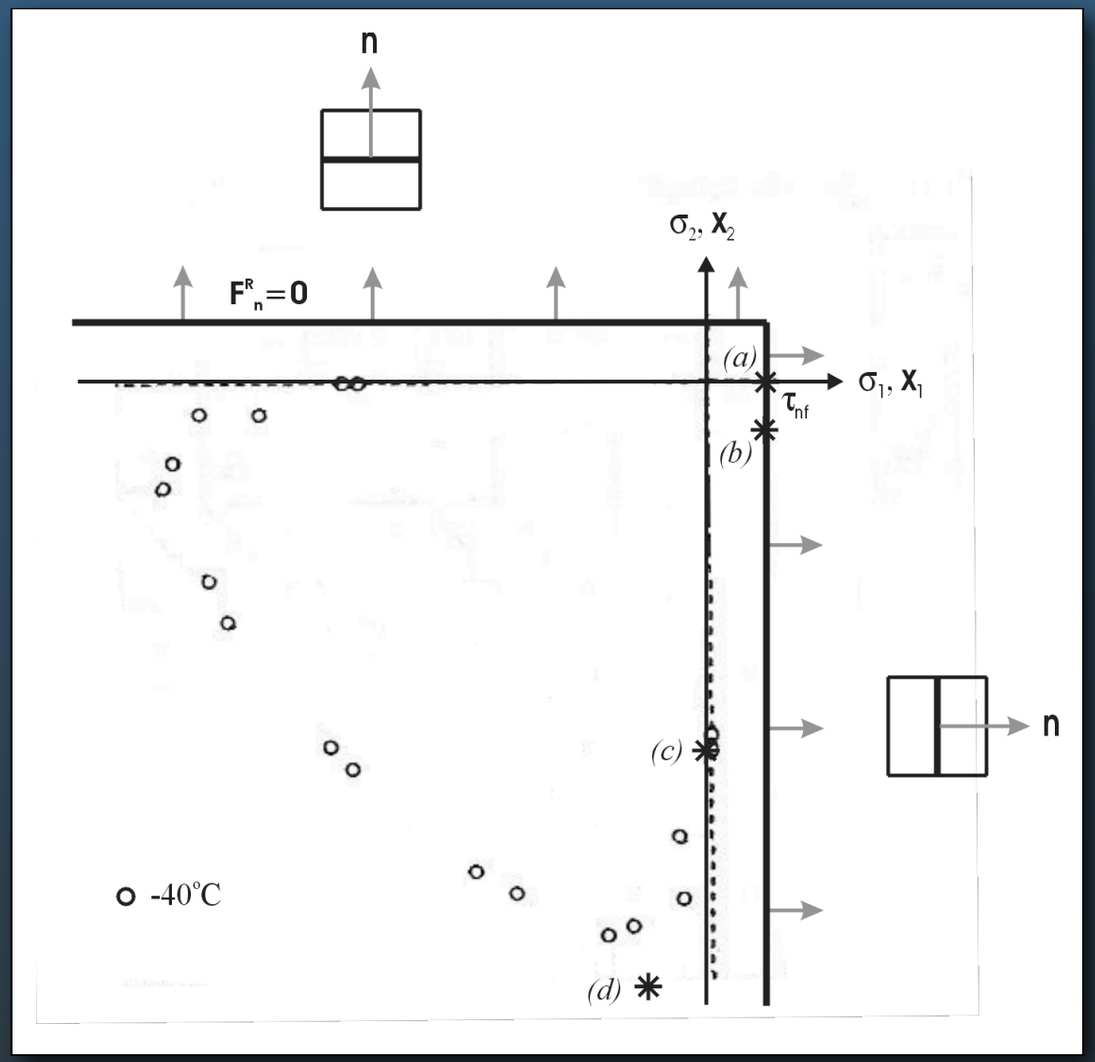


Rankine Criterion

$$F_n^R = \frac{\tau_n}{\tau_{nf}} - 1$$

τ_{nf} = tensile strength



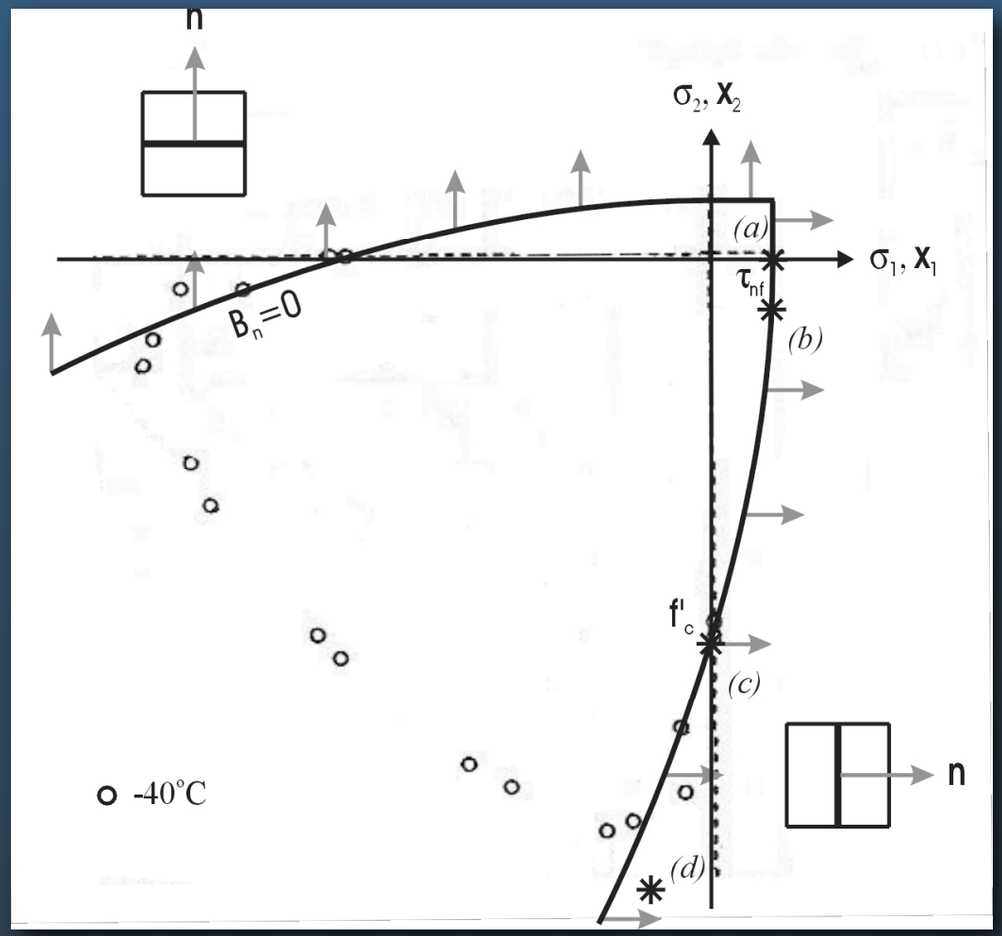


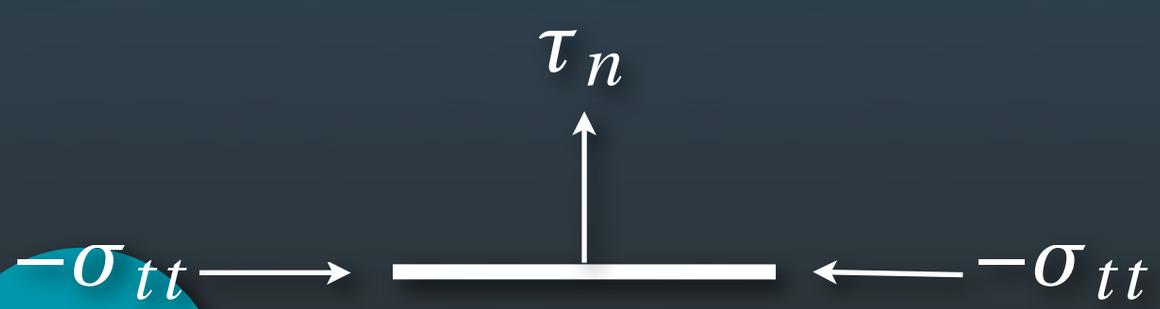
Brittle Decohesion Criterion

$$B_n = \frac{\tau_n}{\tau_{nf}} + \frac{\langle -\sigma_{tt} \rangle^2}{f_c'^2} - 1$$

$$\langle x \rangle = \begin{cases} x, & \text{if } x \geq 0 \\ 0, & \text{if } x < 0 \end{cases}$$

f_c' = compressive strength

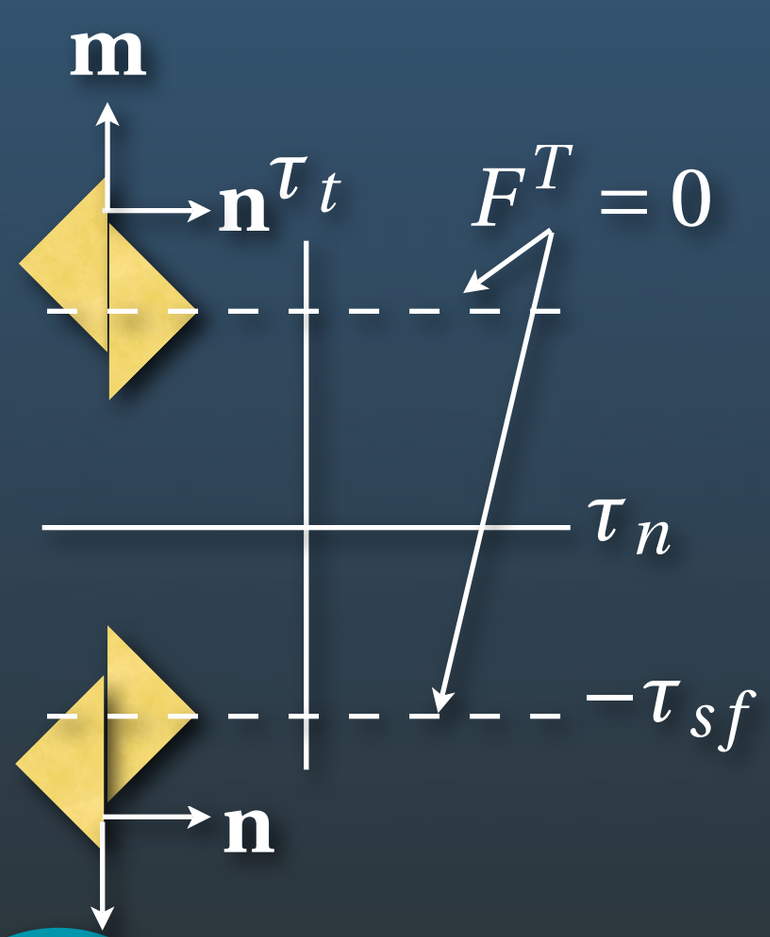


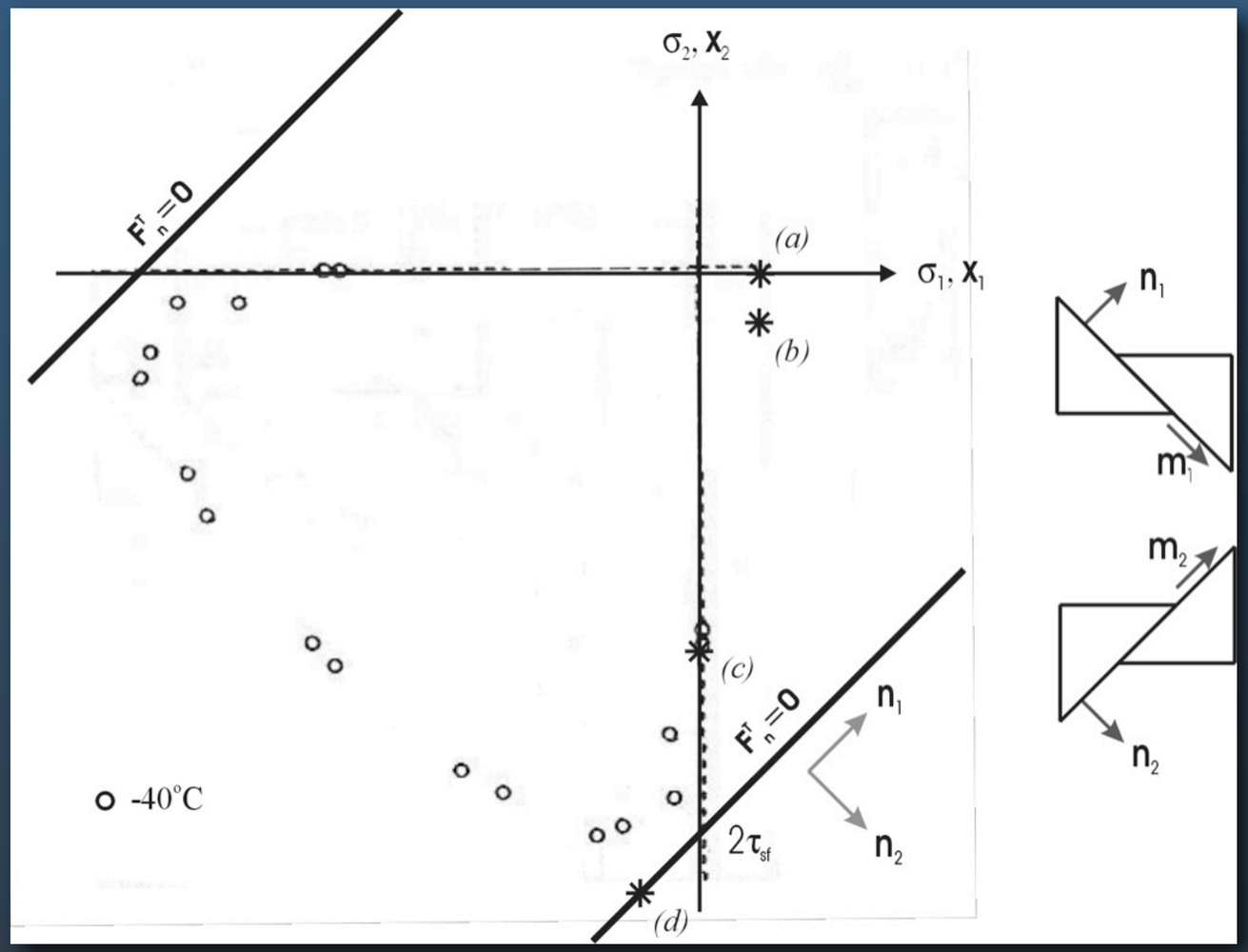


Tresca Criterion

$$F_n^T = \left(\frac{\tau_t}{\tau_{sf}}\right)^2 - 1$$

τ_{sf} = shear strength





Stress at Failure - Failure Initiation - Orientation

Schreyer, H., L. Monday, D. Sulsky, M. Coon, R. Kwok (2006), Elastic-decohesive Constitutive Model for Sea Ice, J. of Geophys. Res., 111, C11S26, doi:10.1029/2005JC003334.

Schulson, E.M. (2001) Brittle failure of ice, Engng. Fract. Mech., 68:1879-1887.

$$F_n = \left(\frac{\tau_t}{s_m \tau_{sf}}\right)^2 + e^{\kappa B_n} - 1$$

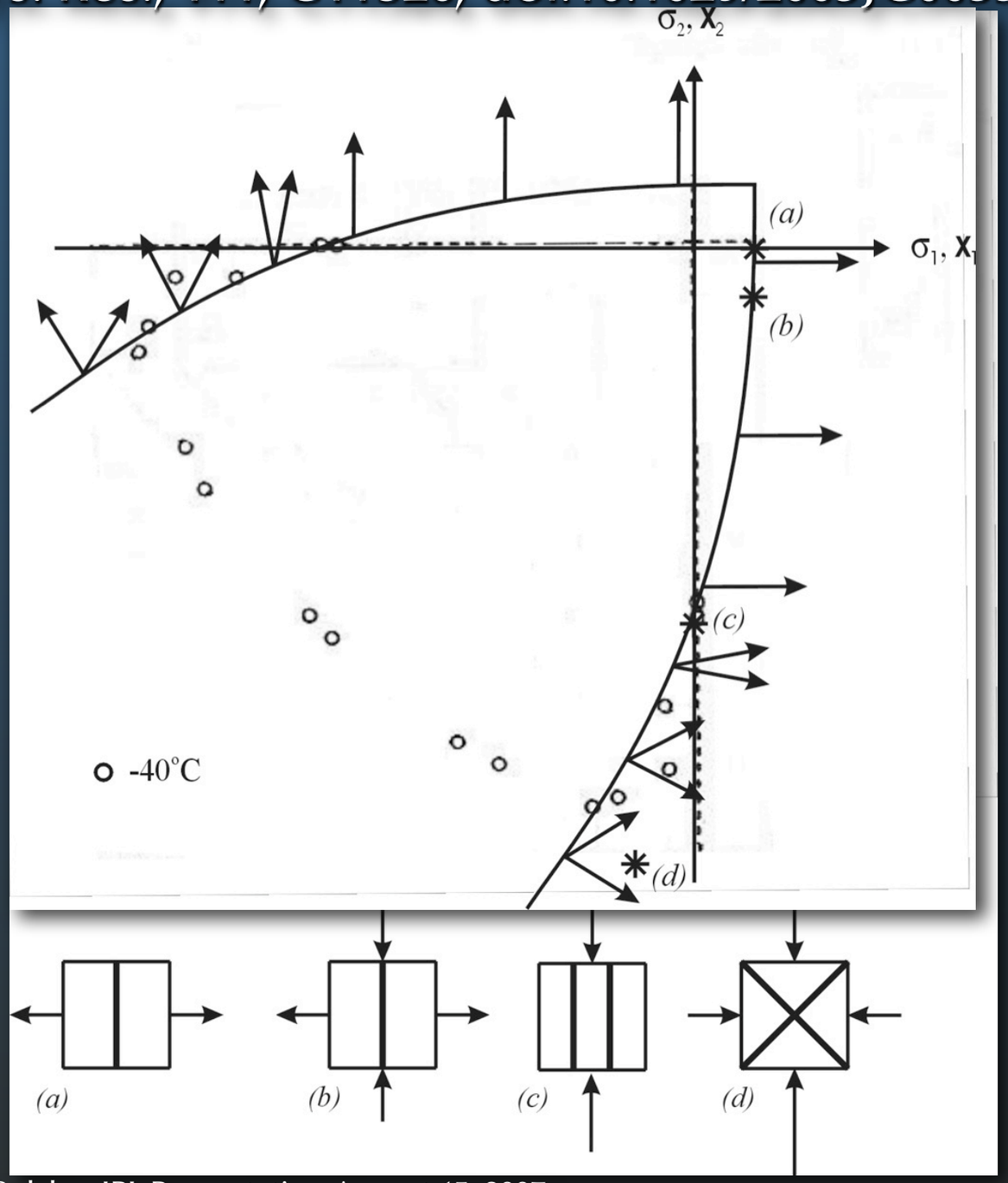
$$B_n = \frac{\tau_n}{\tau_{nf}} + \frac{\langle -\sigma_{tt} \rangle^2}{f_c'^2} - 1$$

τ_{nf} = tensile strength

τ_{sf} = shear strength

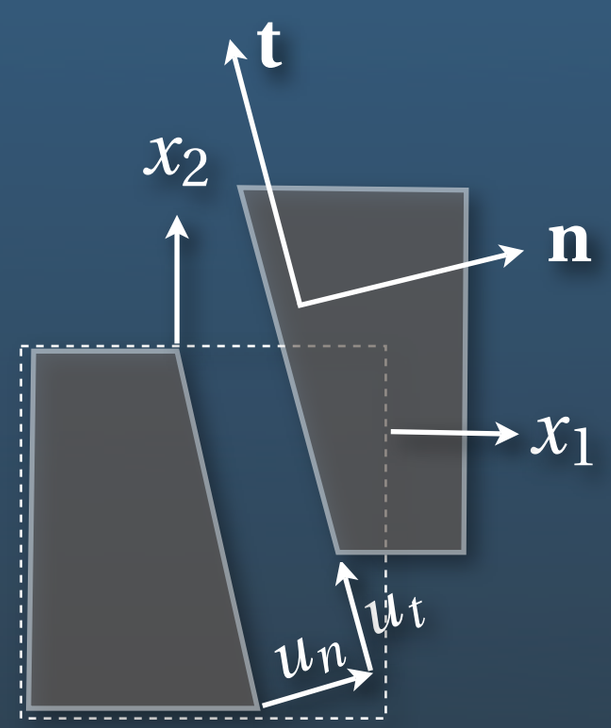
f_c' = compressive strength

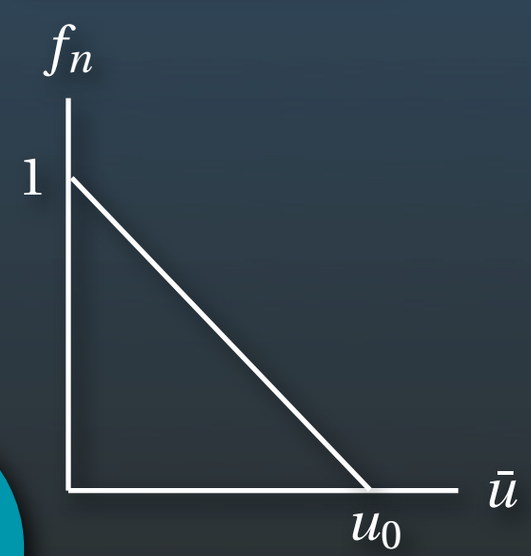
s_m = shear magnification



Failure Evolution

As decohesion occurs material becomes weaker.





$$[\mathbf{u}] = u_n \mathbf{n} + u_t \mathbf{t}$$

$$\bar{u} = u_n$$

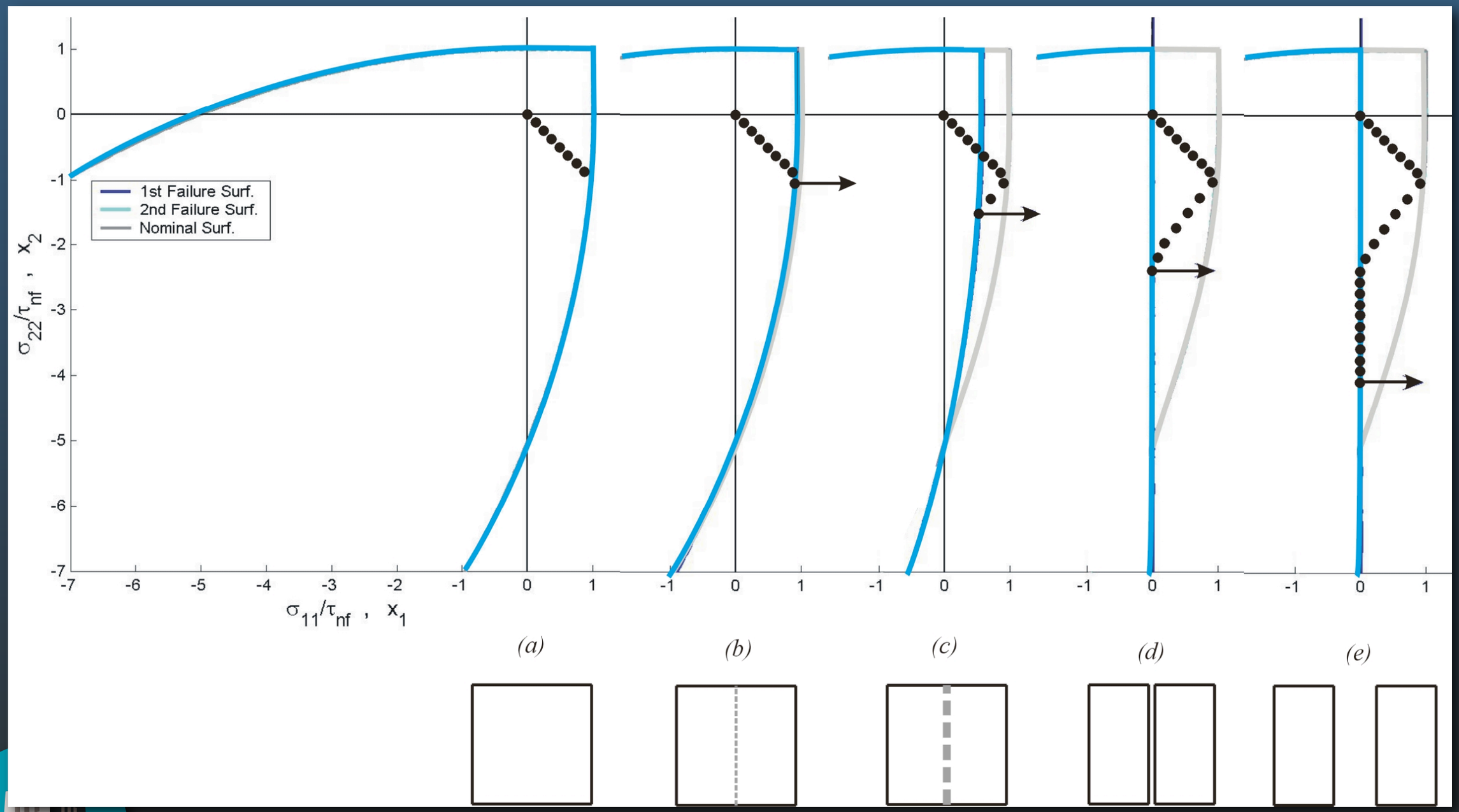
$$f_n = \left\langle 1 - \frac{\bar{u}}{u_0} \right\rangle$$

$$B_n = \frac{\tau_n}{\tau_{nf}} + \boxed{f_n} \left[\frac{\langle -\sigma_{tt} \rangle^2}{f_c'^2} - 1 \right]$$

$$F_n = \left(\frac{\tau_t}{s_m \tau_{sf}} \right)^2 + e^{\kappa B_n} - 1$$

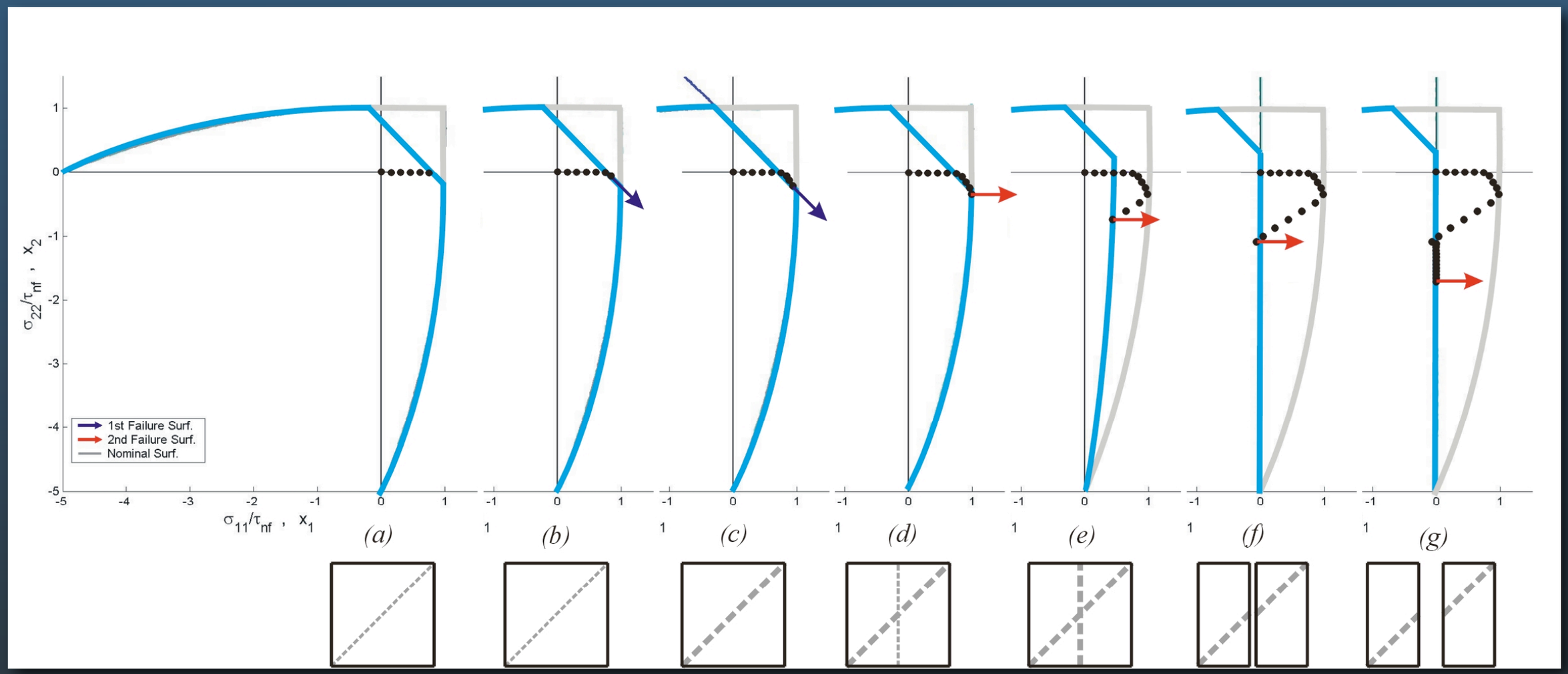
Failure Evolution

$$e_1 = -e_2$$

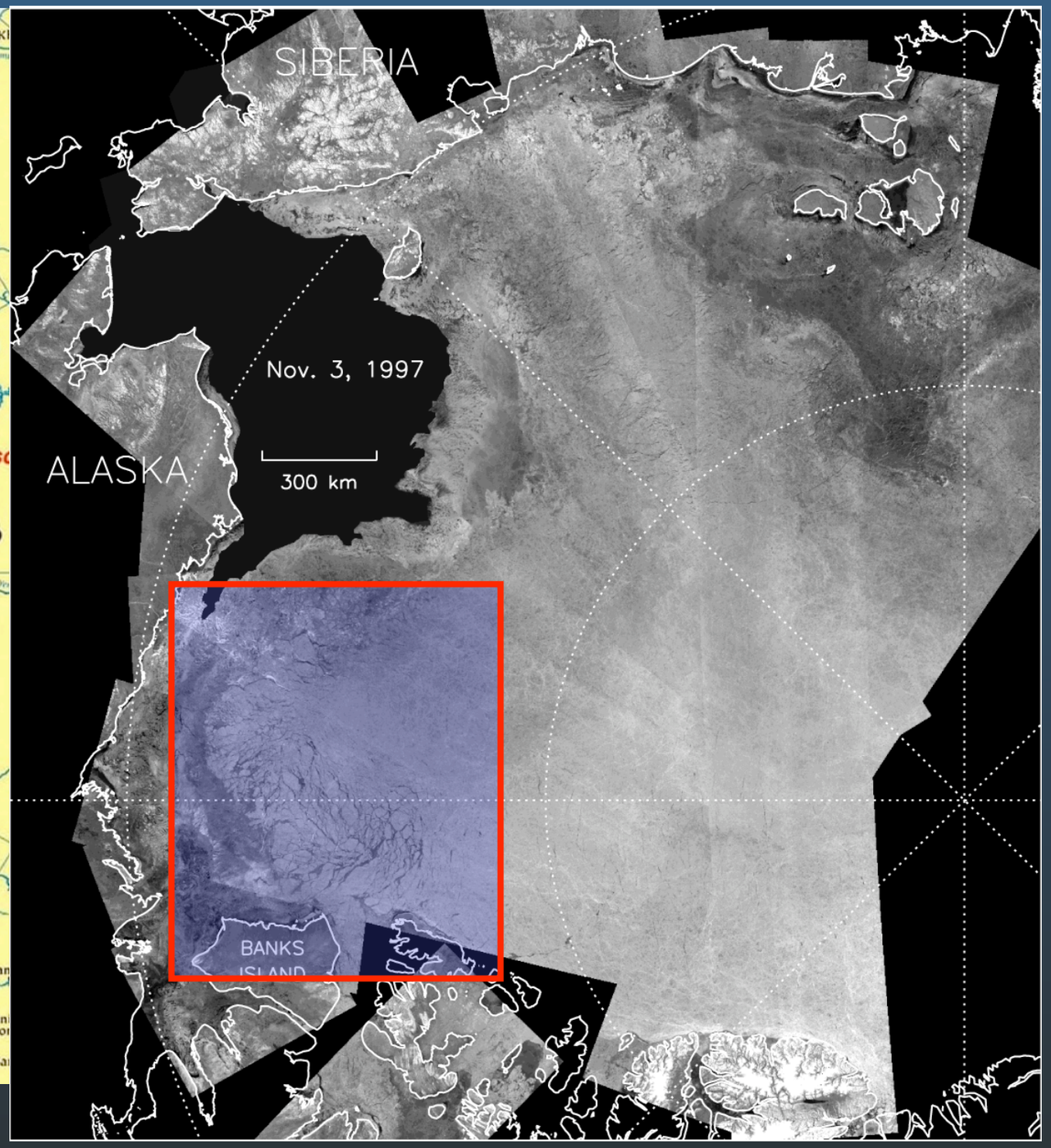


Initialized Weak Plane

$f_n = 0.5$ $\theta = 45^\circ$



Arctic Sea Ice



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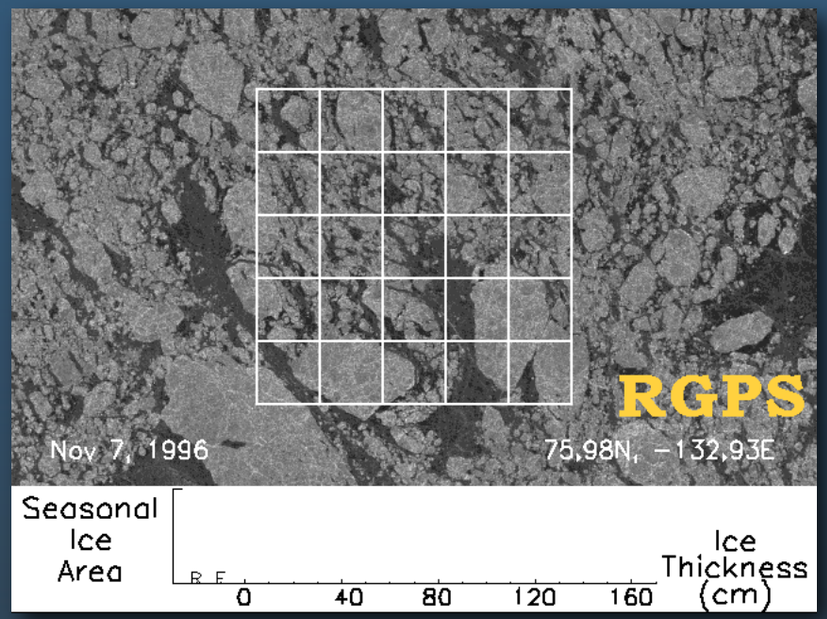
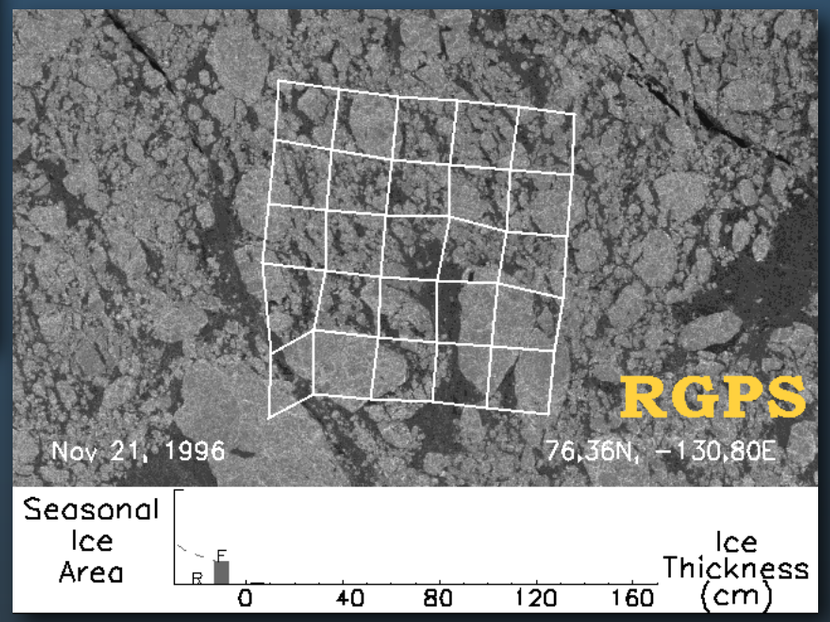
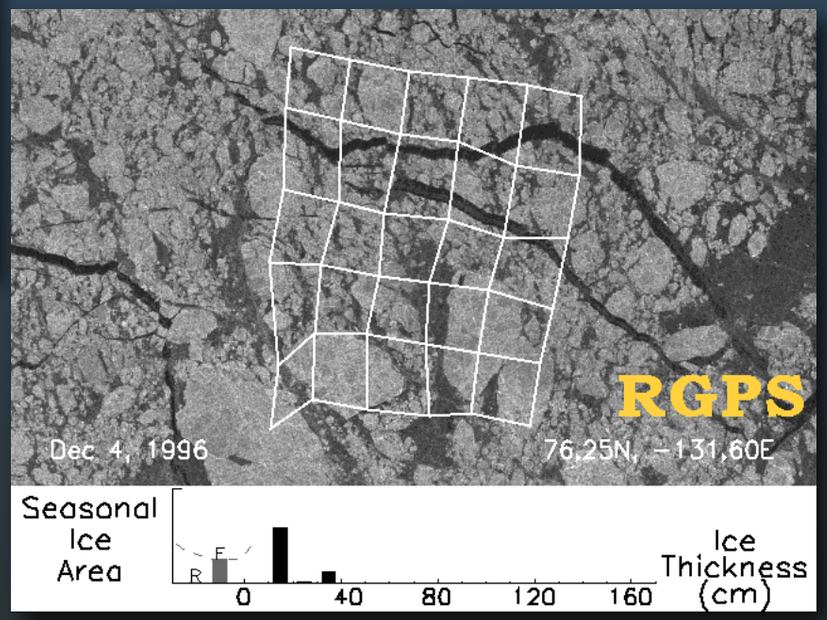
RGPS

Radarsat Geophysical Processor System at JPL







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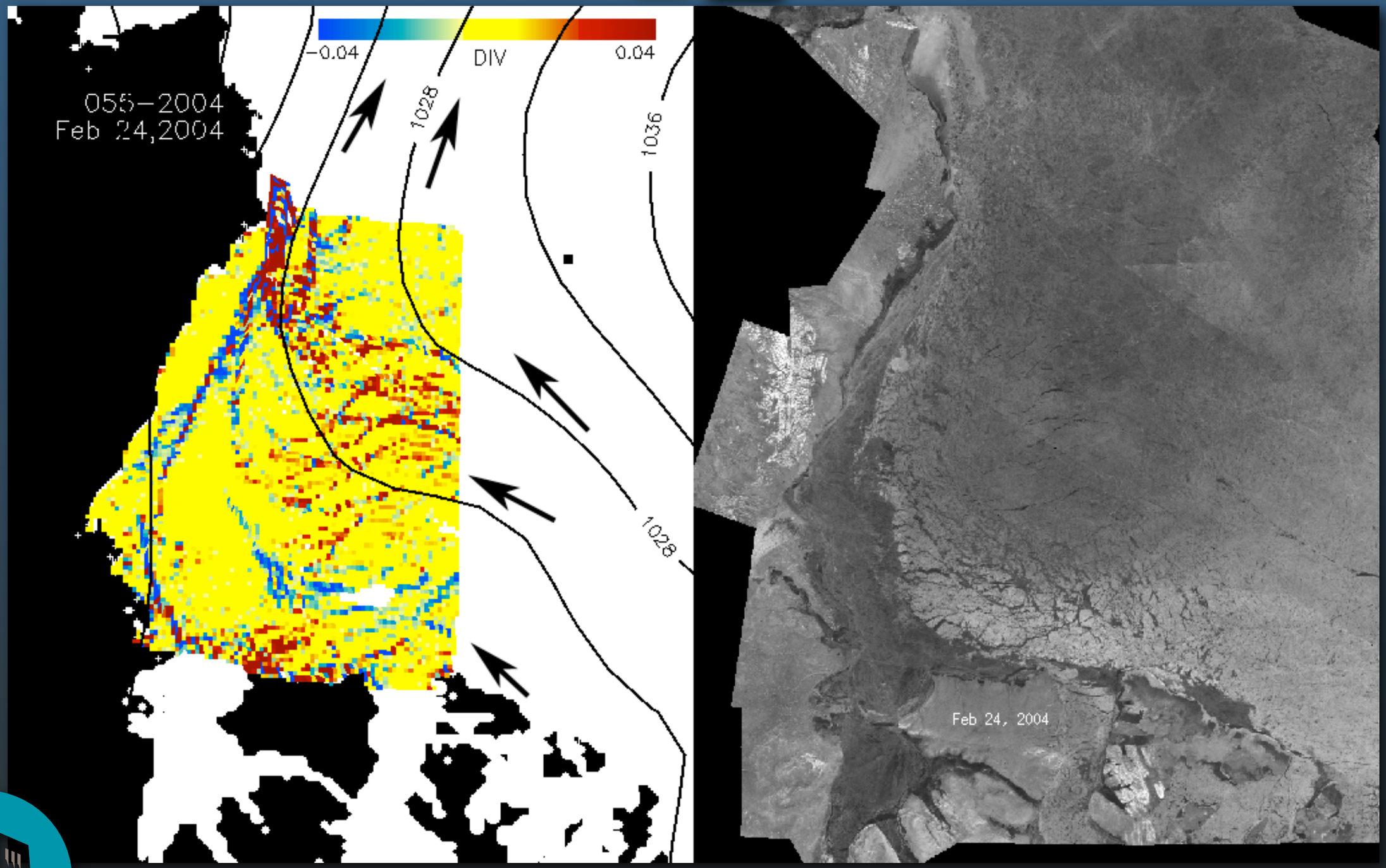
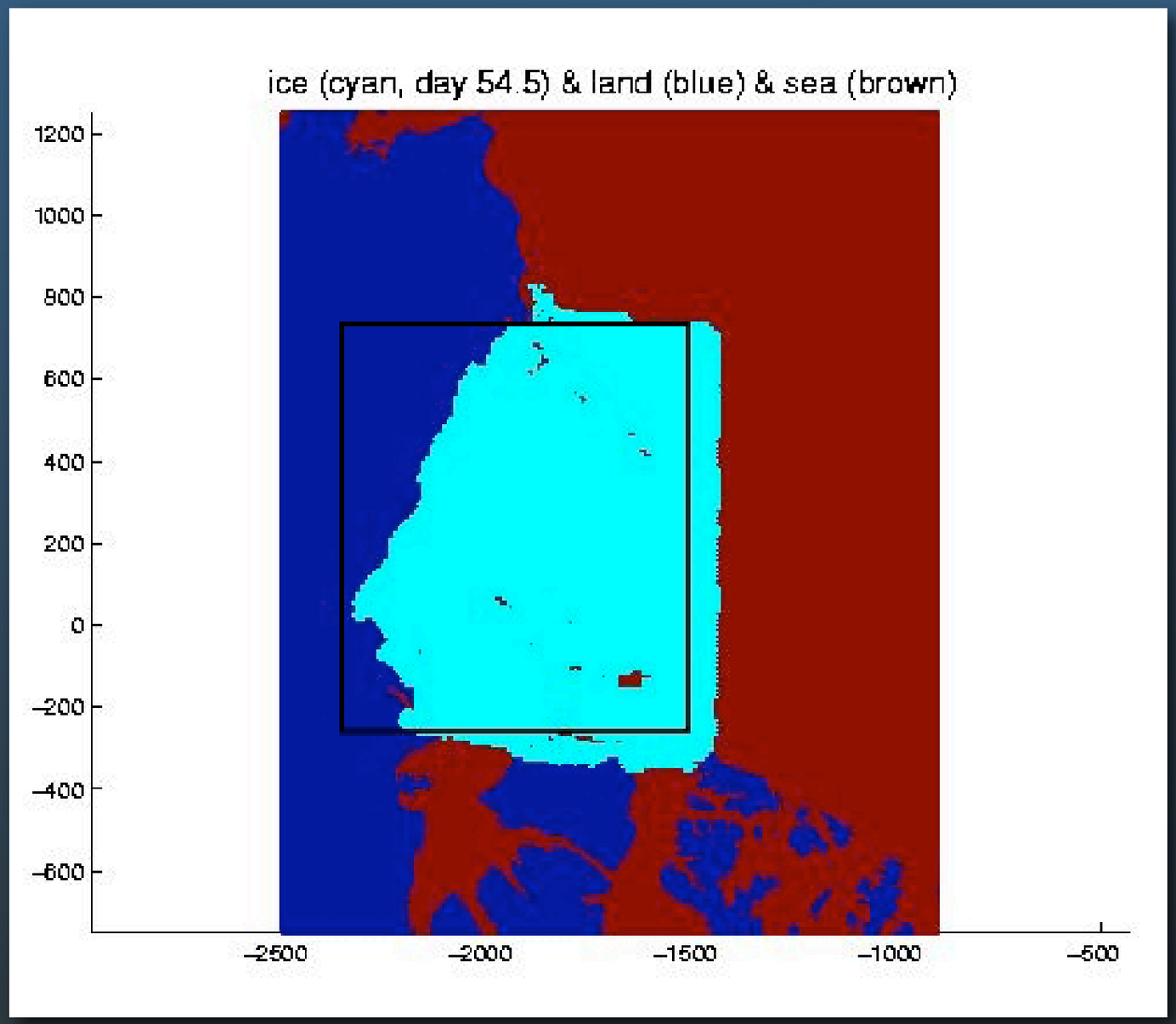
Divergence



Problem Set Up

Simulate 16 days in Feb/Mar, 2004

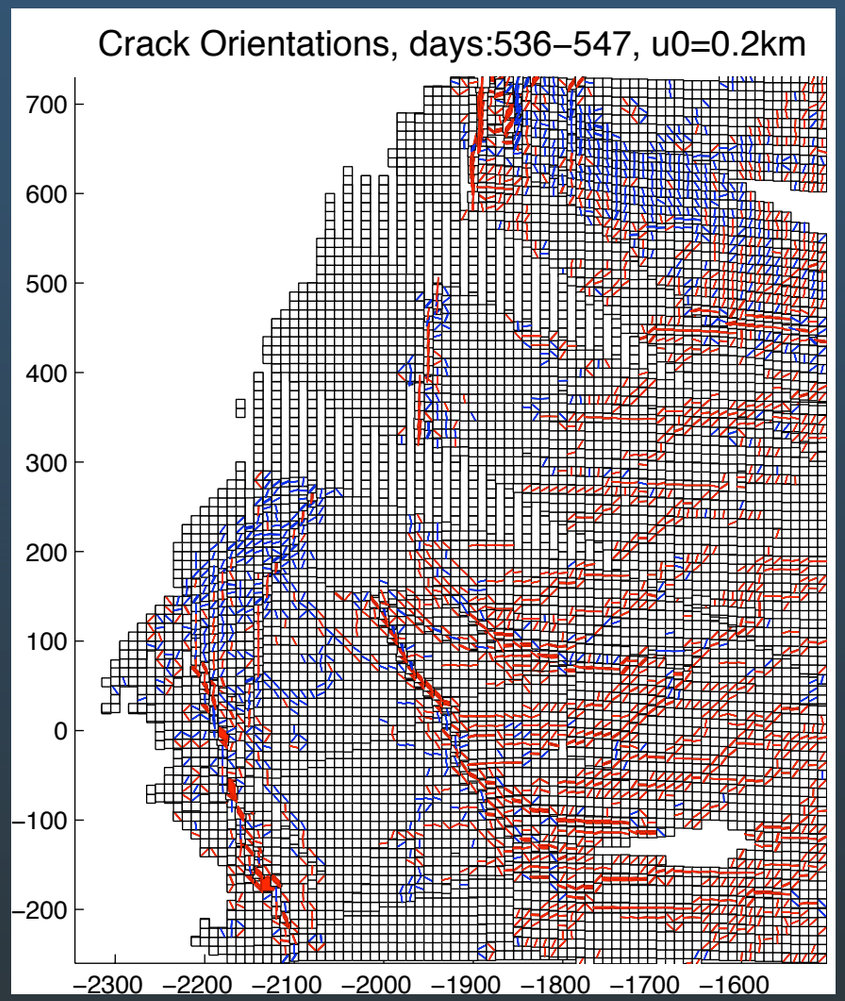


Set up:

- 10 km square background grid
- 4 material points per element
- rigid material points for land
- include wind, ocean, and Coriolis forces
- Right, top, bottom boundary conditions from RGPS displacements

Initialization

Use a kinematic analysis of satellite data to find existing leads



$E = 1$ MPa

$\nu = 0.36$

$\tau_{nf} = 25$ KPa

$\tau_{sf} = 15$ KPa

$f'_c = 125$ KPa

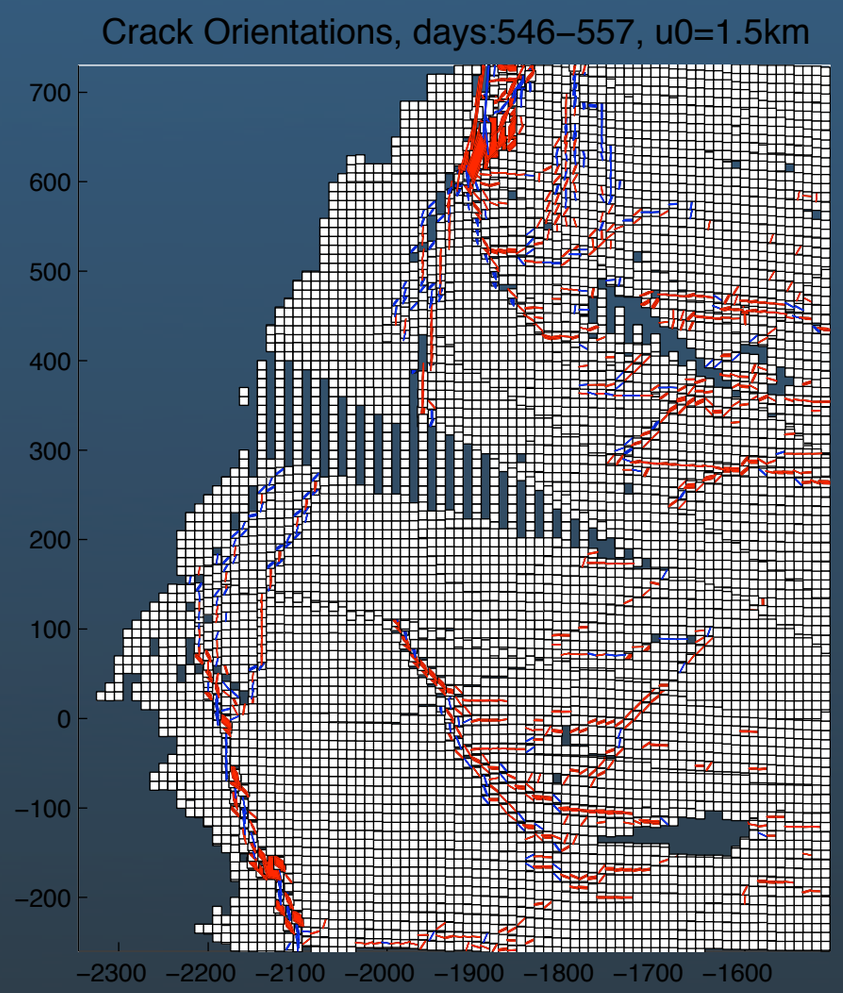
$u_0 = 400$ m

$s_m = 4$

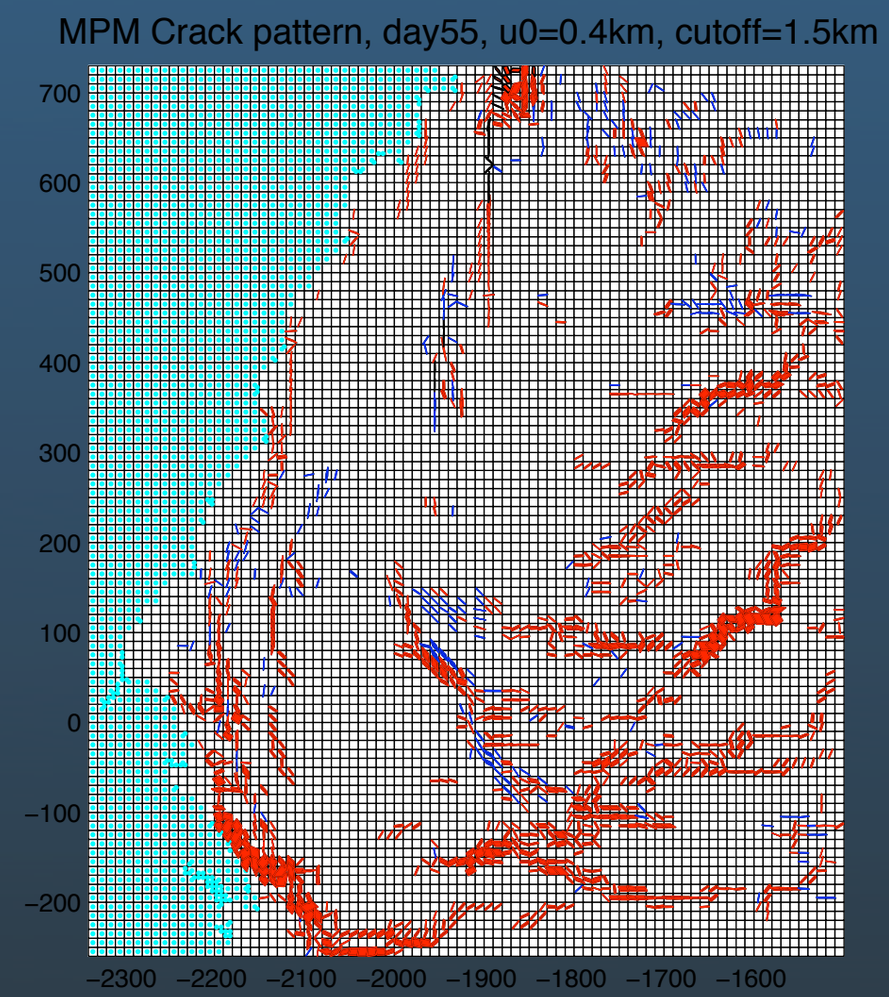
Day 54 (Feb. 23)

Fracture Patterns in the Beaufort

day 55



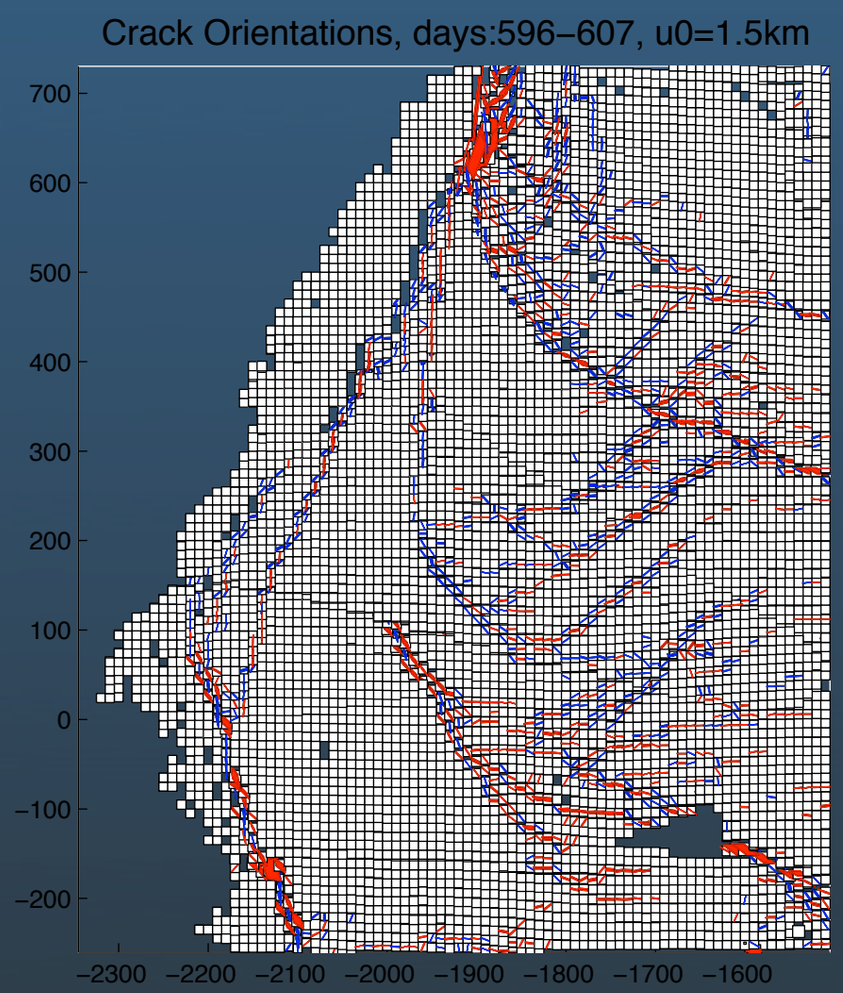
Observation



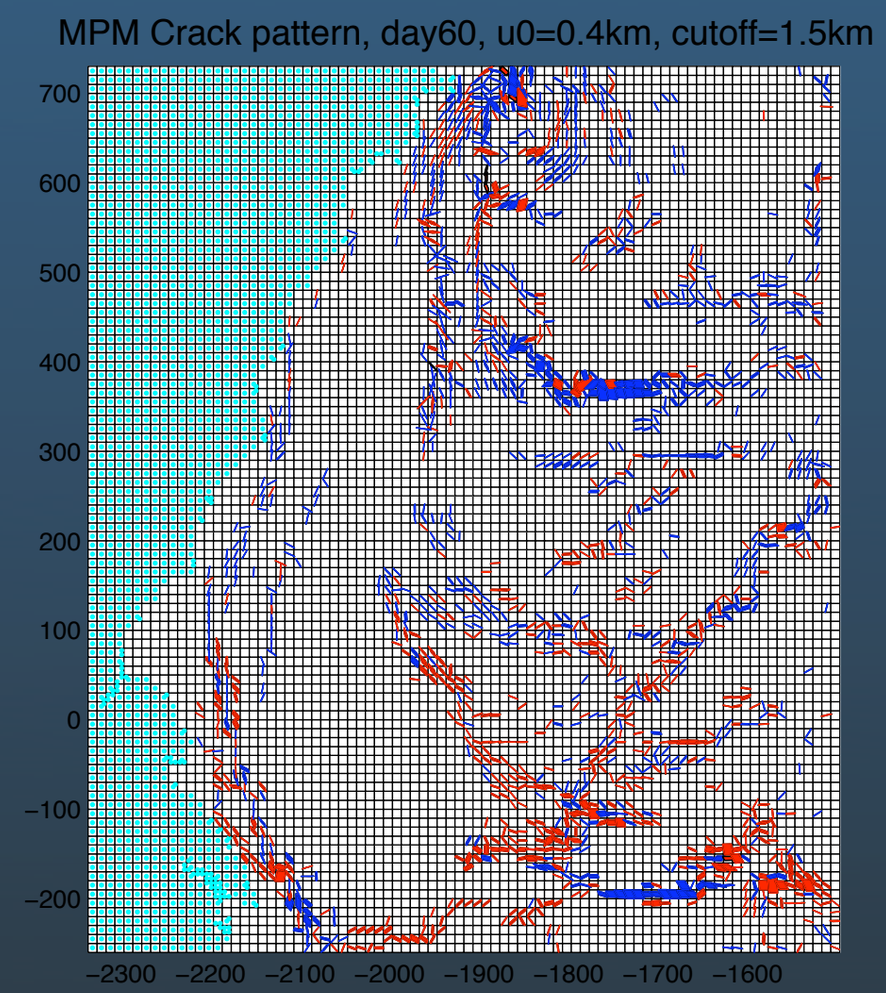
Simulation

Fracture Patterns in the Beaufort

day 60



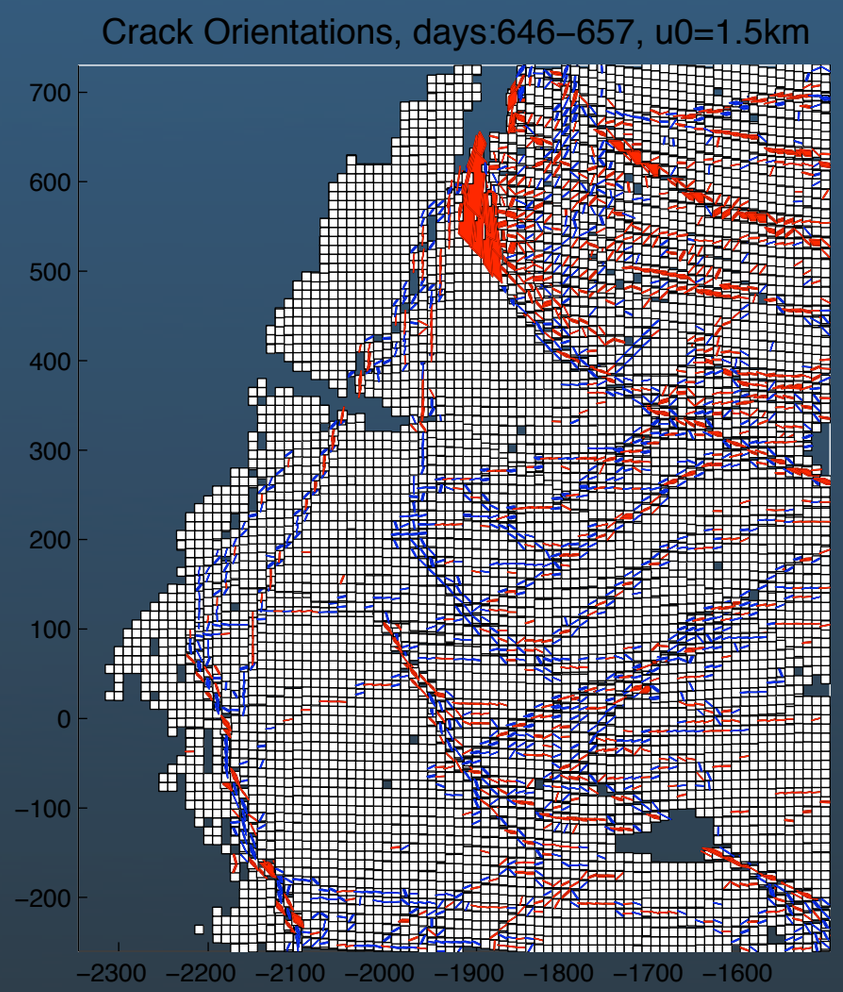
Observation



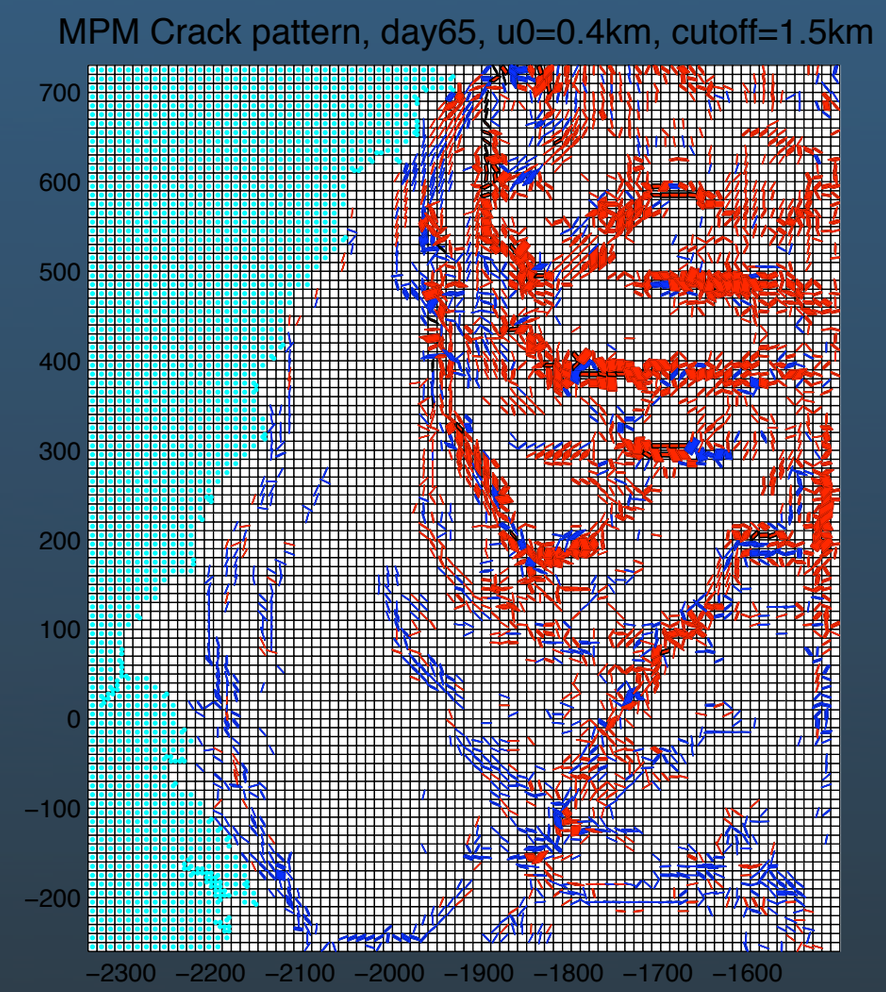
Simulation

Fracture Patterns in the Beaufort

day 65



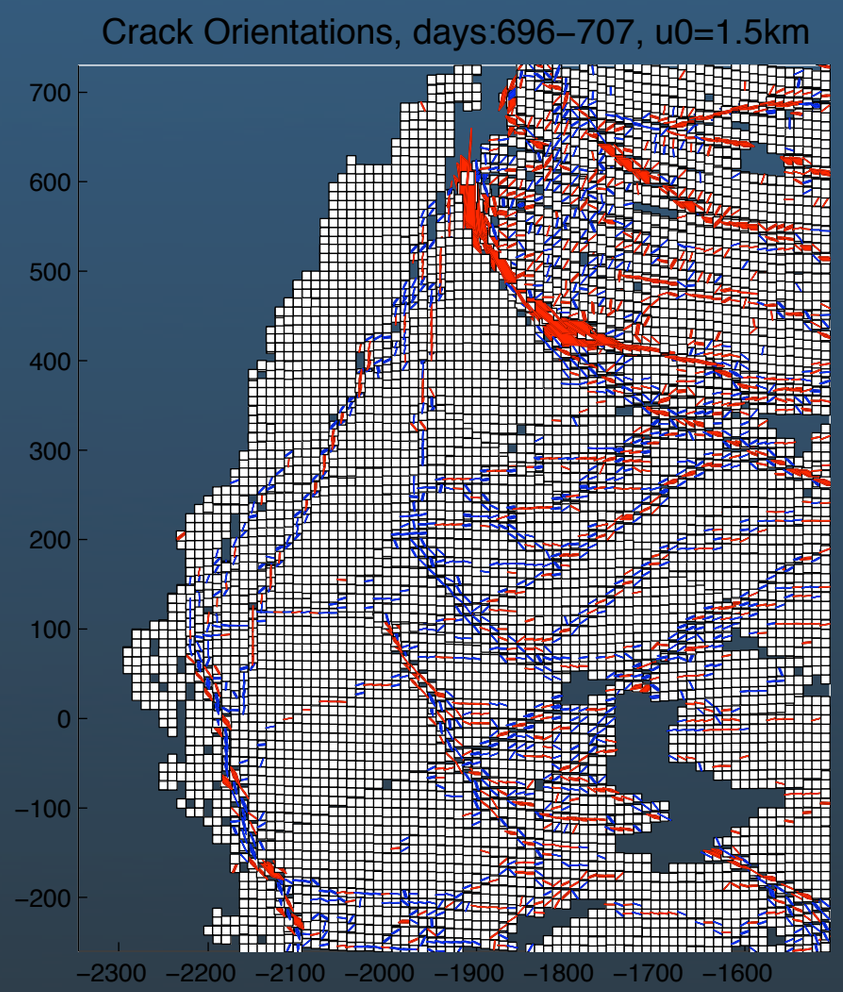
Observation



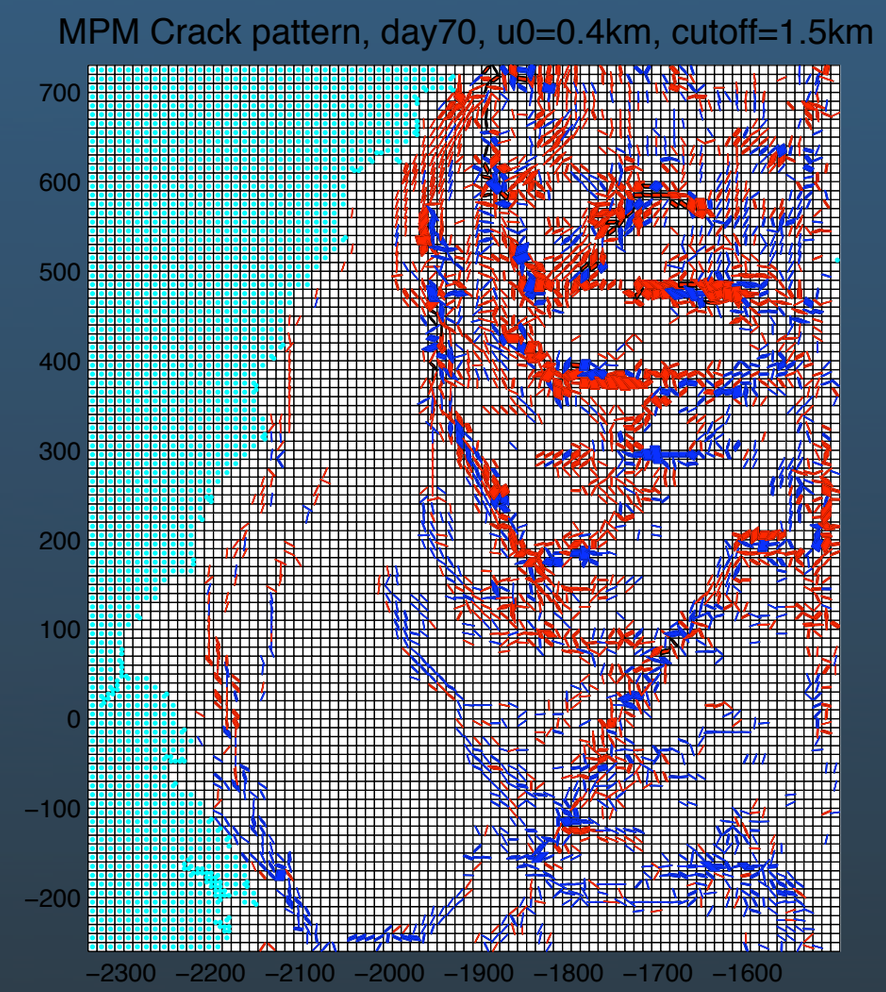
Simulation

Fracture Patterns in the Beaufort

day 70



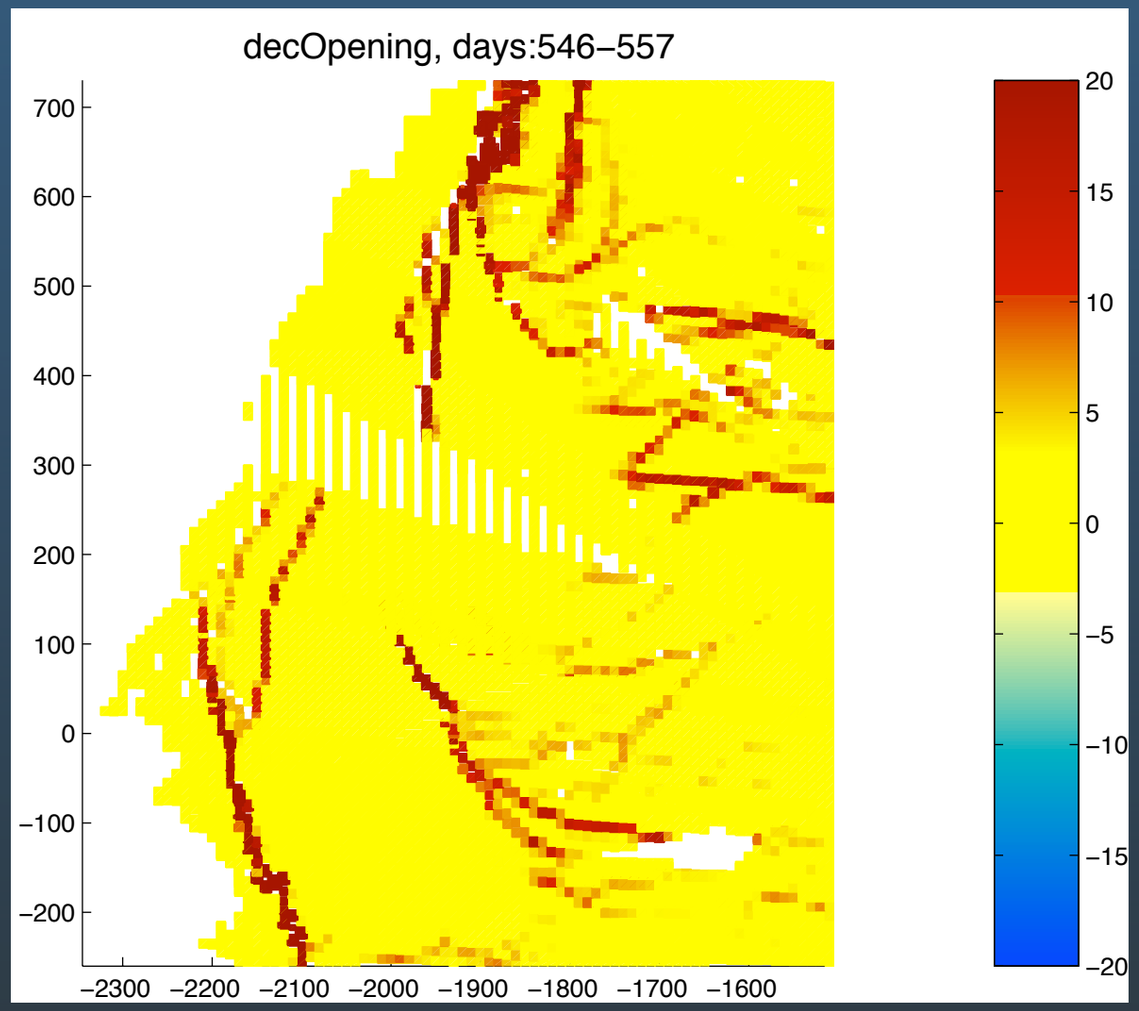
Observation



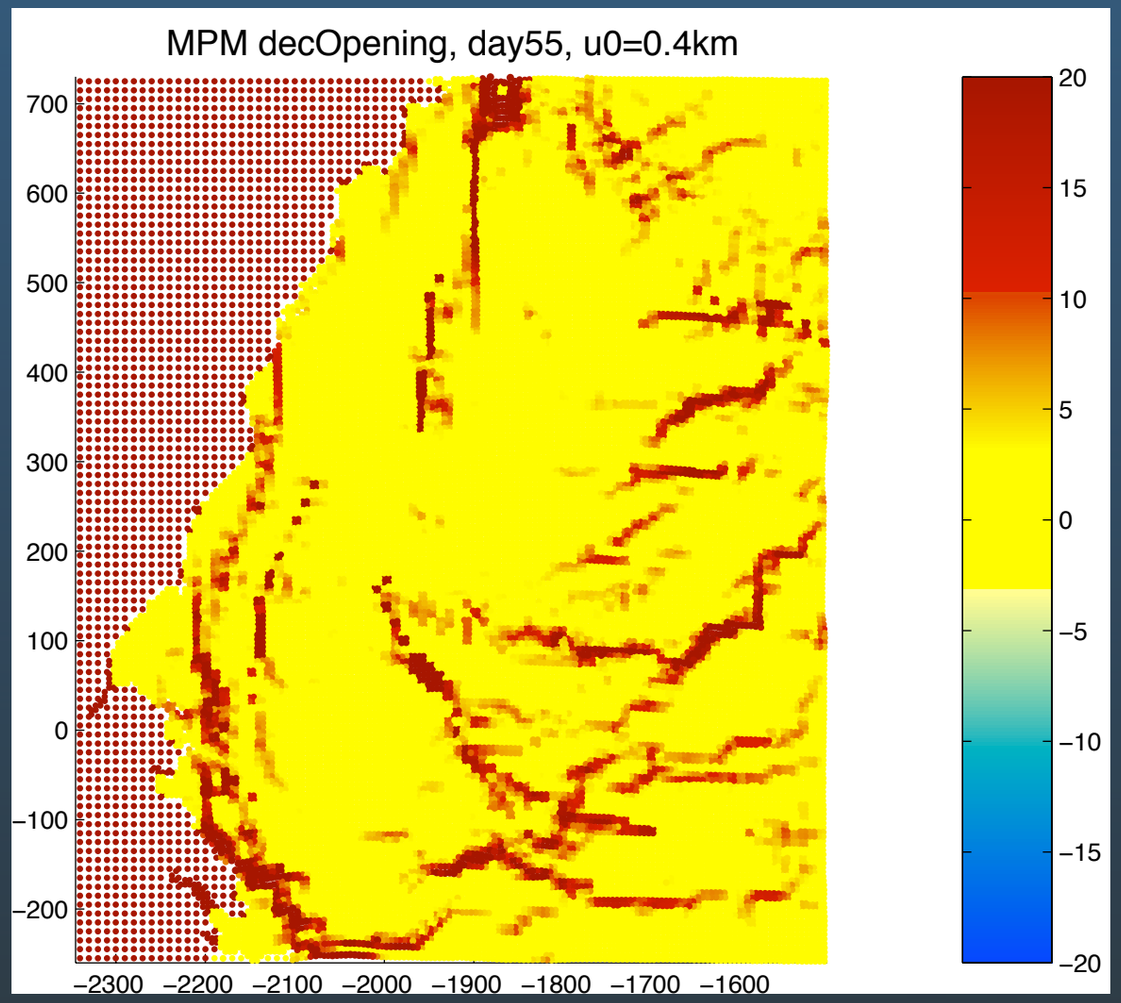
Simulation

Fracture Patterns in the Beaufort

day 55



Observation



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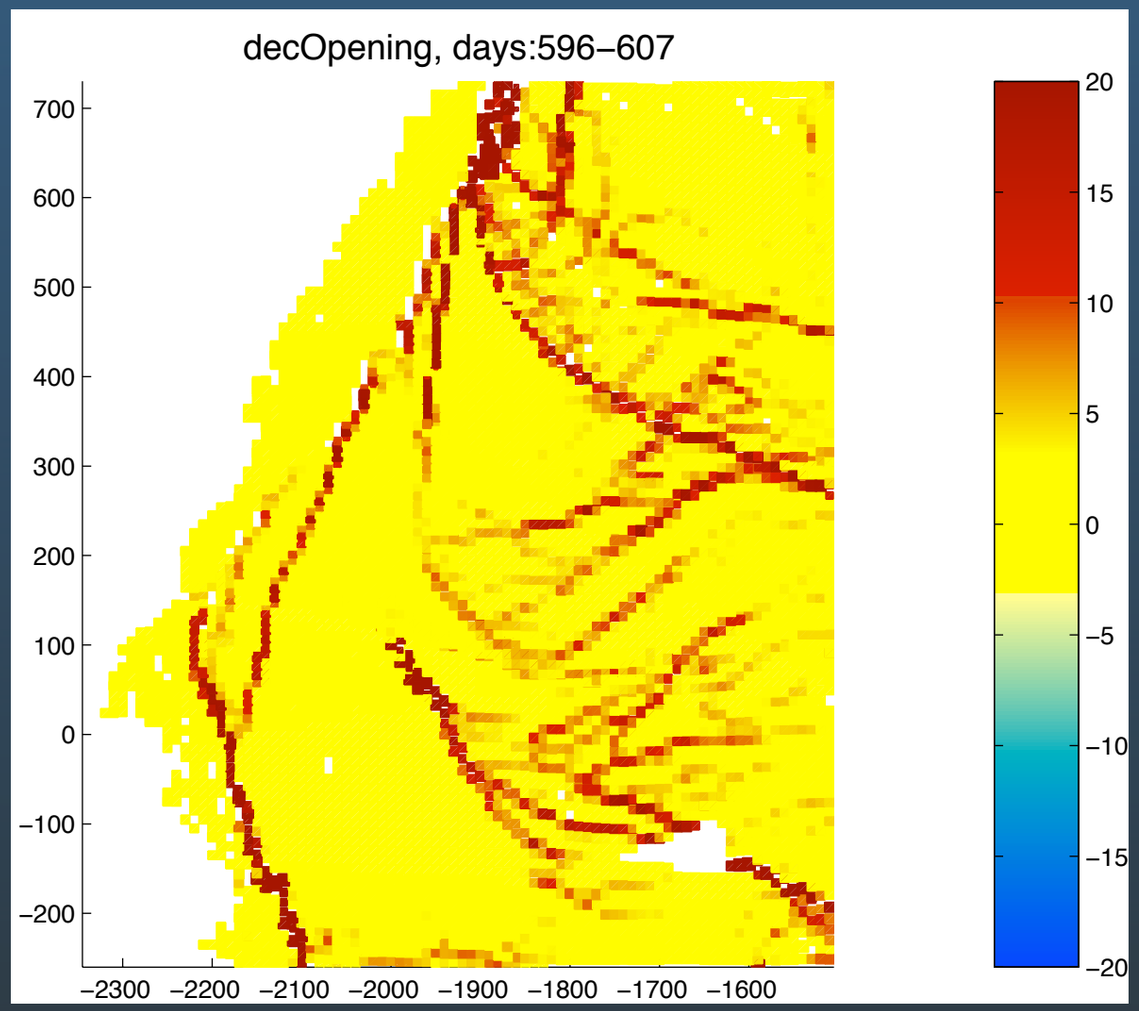
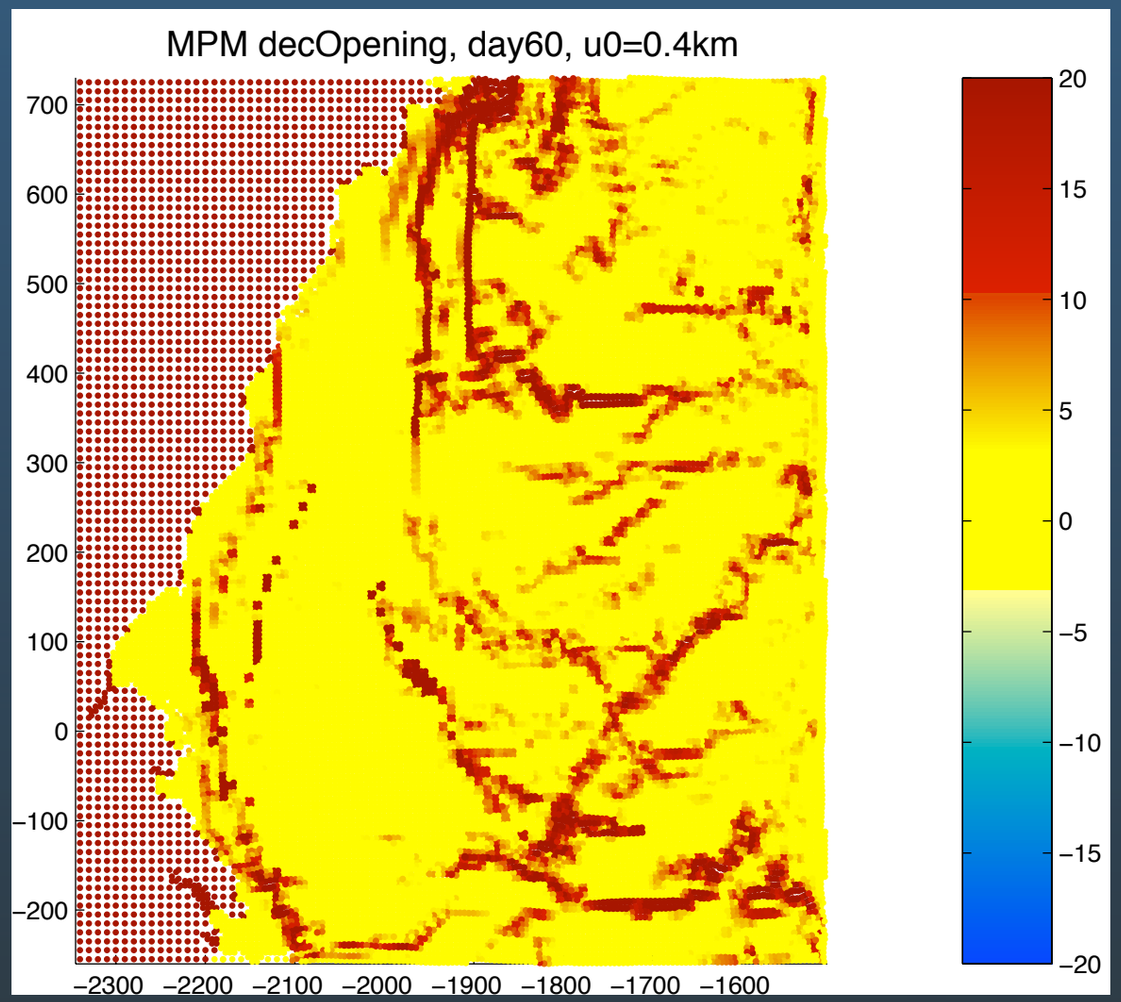
Fracture Patterns in the Beaufort

day 60



Observation



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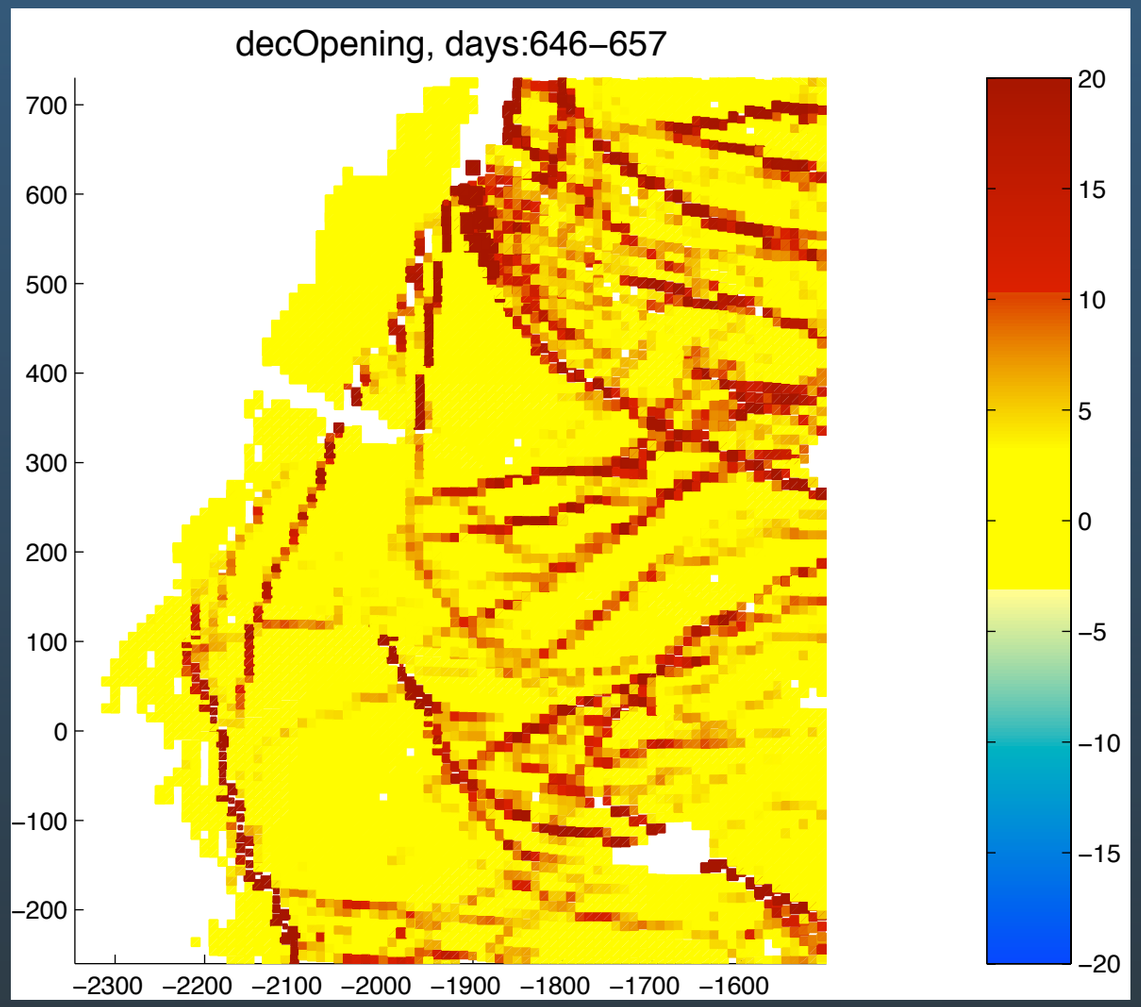
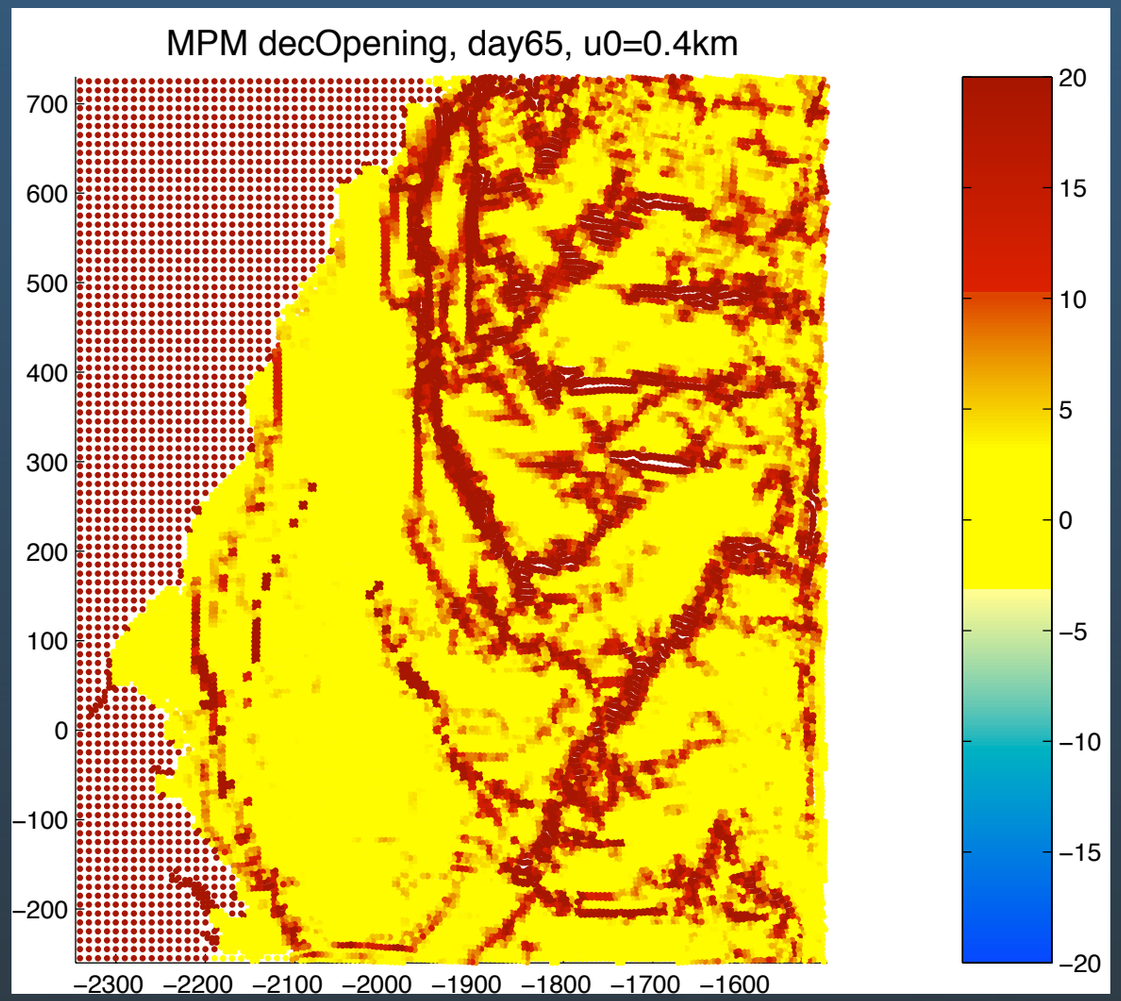
Fracture Patterns in the Beaufort

day 65



Observation



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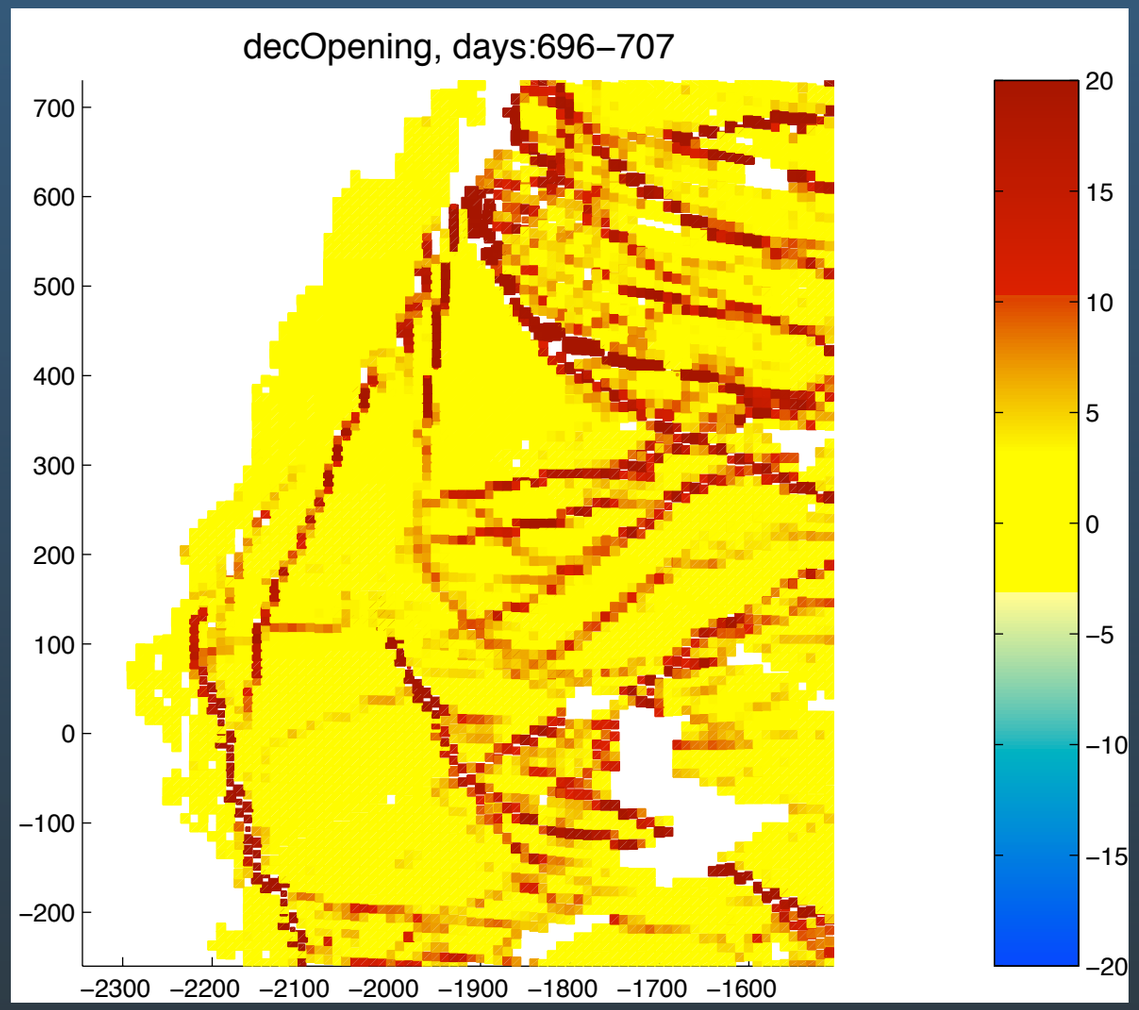
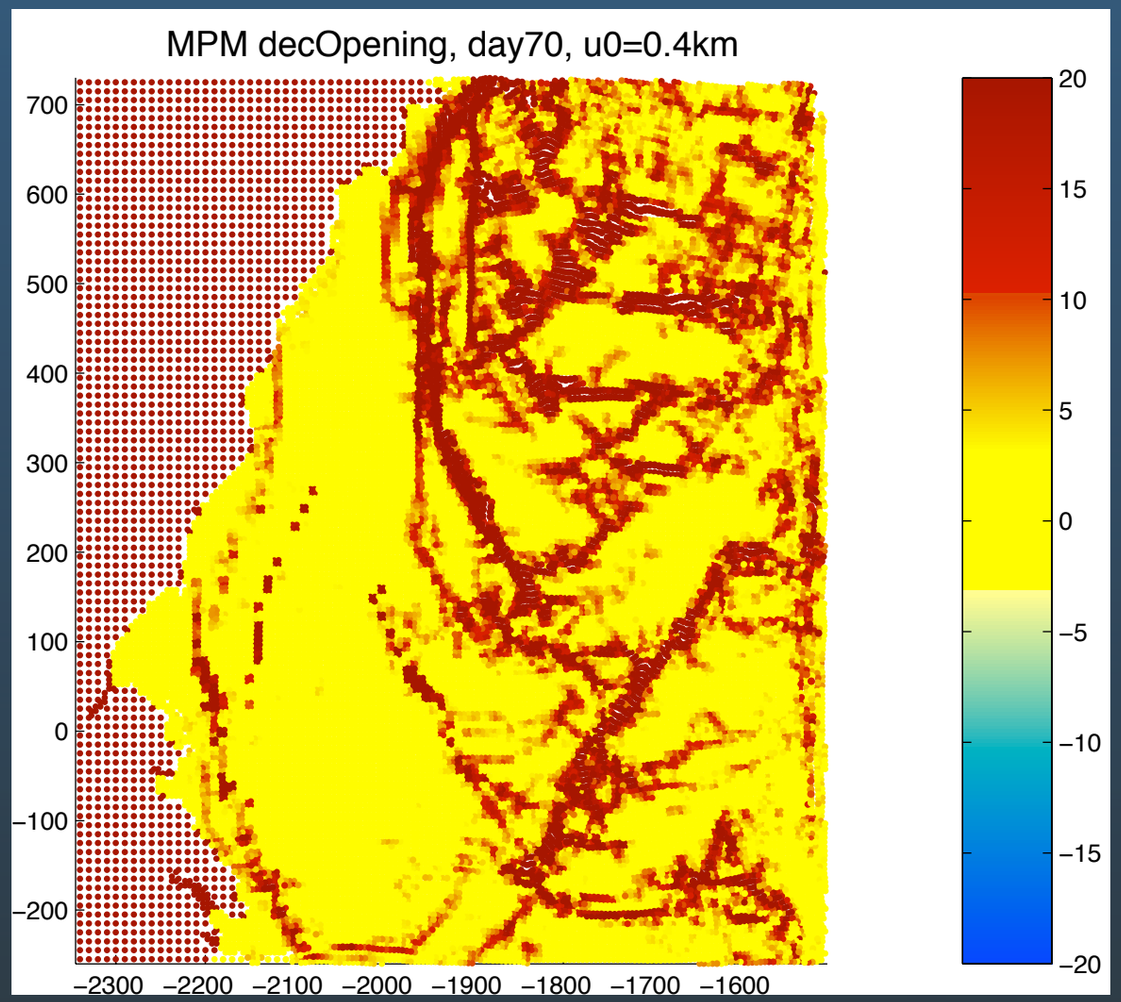
Fracture Patterns in the Beaufort

day 70



Observation



Simulation

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Summary

Features of MPM:

- Handles advection naturally
- Supports large deformations and fracture

Features of elastic-decohesive model:

- Stress state at which leads initiate
- Orientation of lead at initiation
- Evolution of lead (softening)
- Existing material weakness
- Implemented in plasticity framework