Observing world ocean circulation from global compilation of current meters.

Evidence of entropic forcing?

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Arctic Ocean Models Intercomparison Project (AOMIP):

Compare models, T and S are simple. Average, make heat or "freshwater" storage, etc. .... But what to do about **V**?

Define "topostrophy"

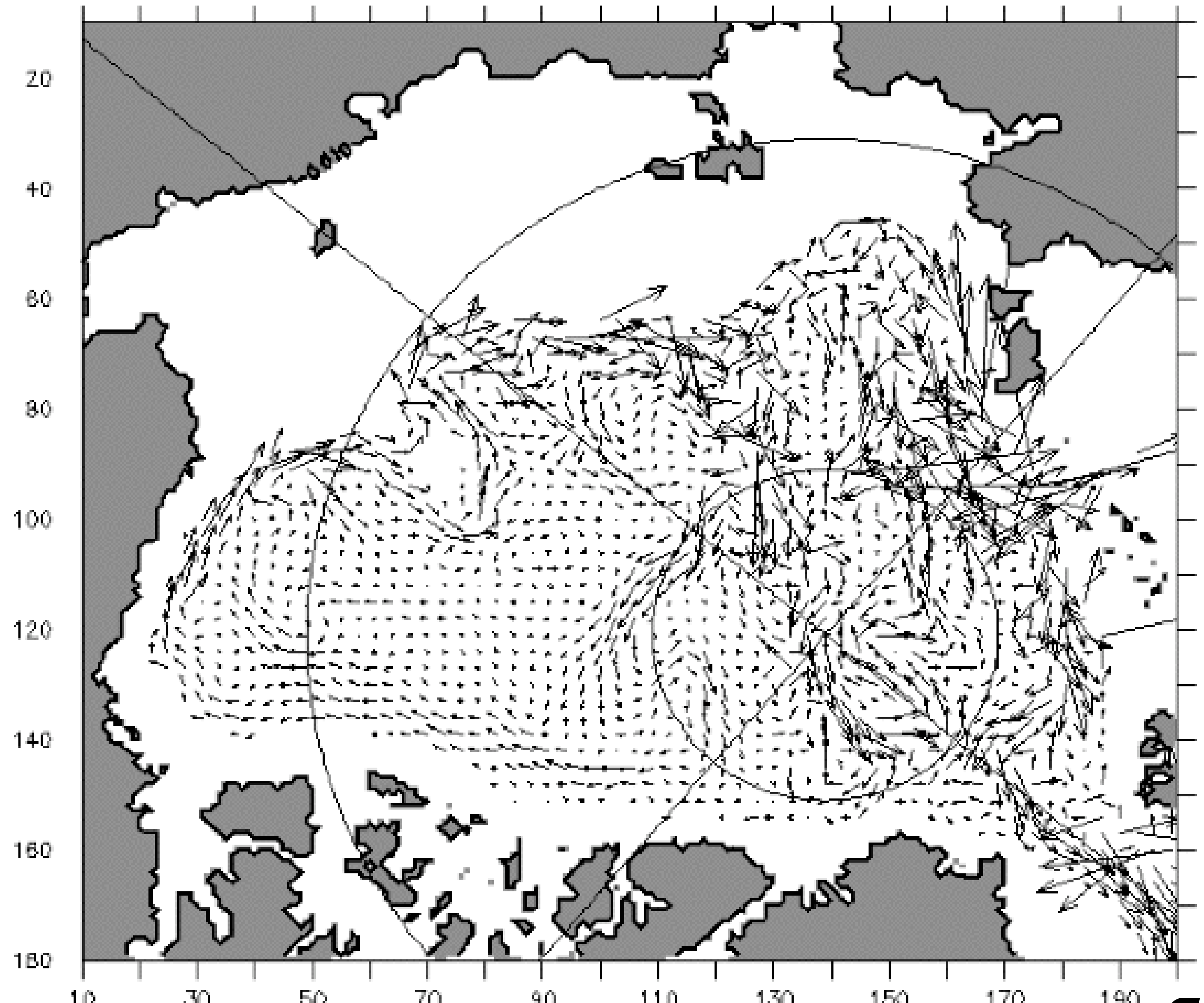
$\tau \equiv \mathbf{f} \times \mathbf{V} \cdot \nabla D$, a scalar that averages like T or S.

Normalize

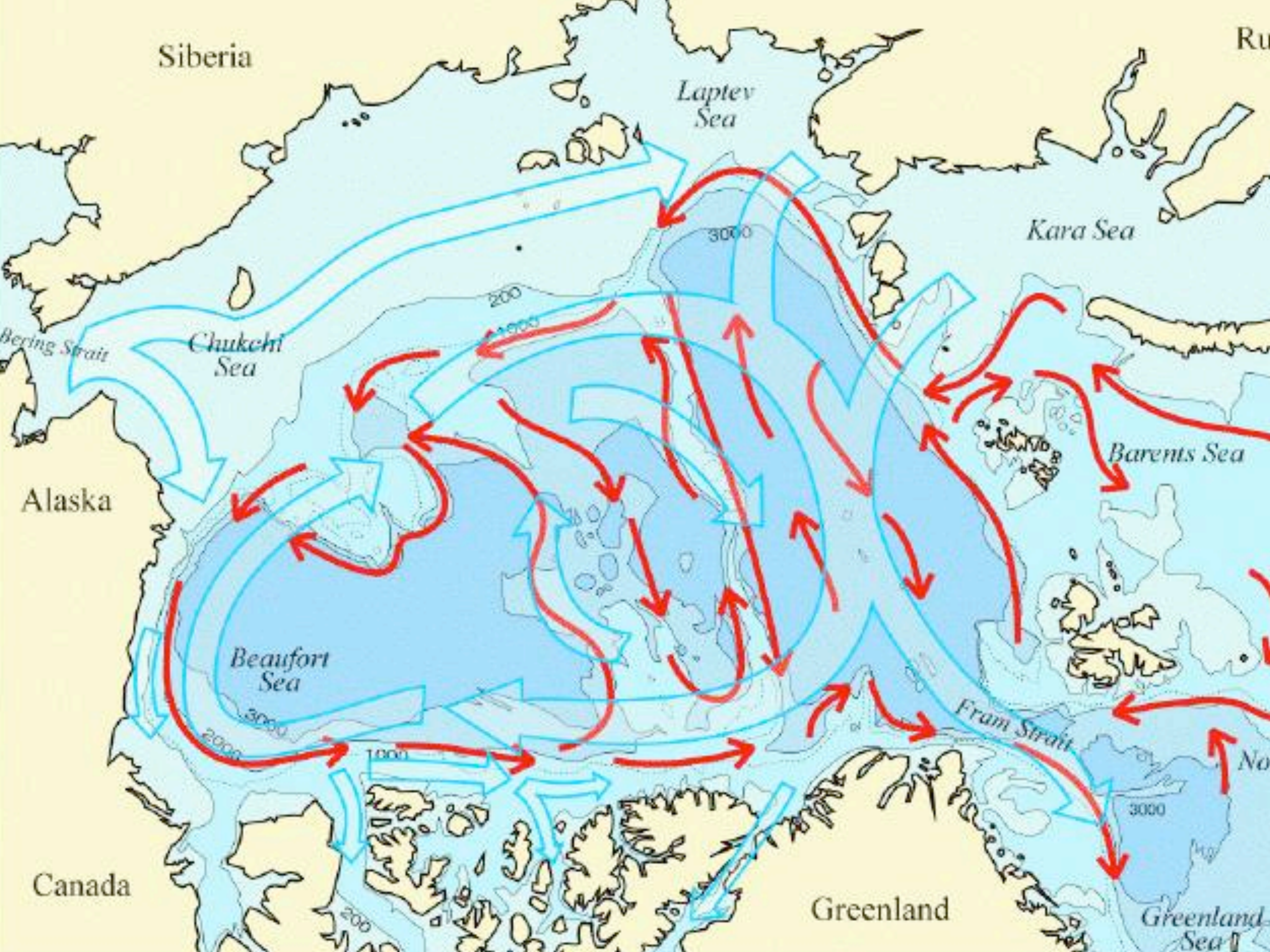
$$\tau \equiv \frac{\langle \mathbf{f} \times \mathbf{V} \cdot \nabla D \rangle}{\sqrt{\left\langle |\mathbf{f} \times \mathbf{V}|^2 \right\rangle \left\langle |\nabla D|^2 \right\rangle}}$$

then

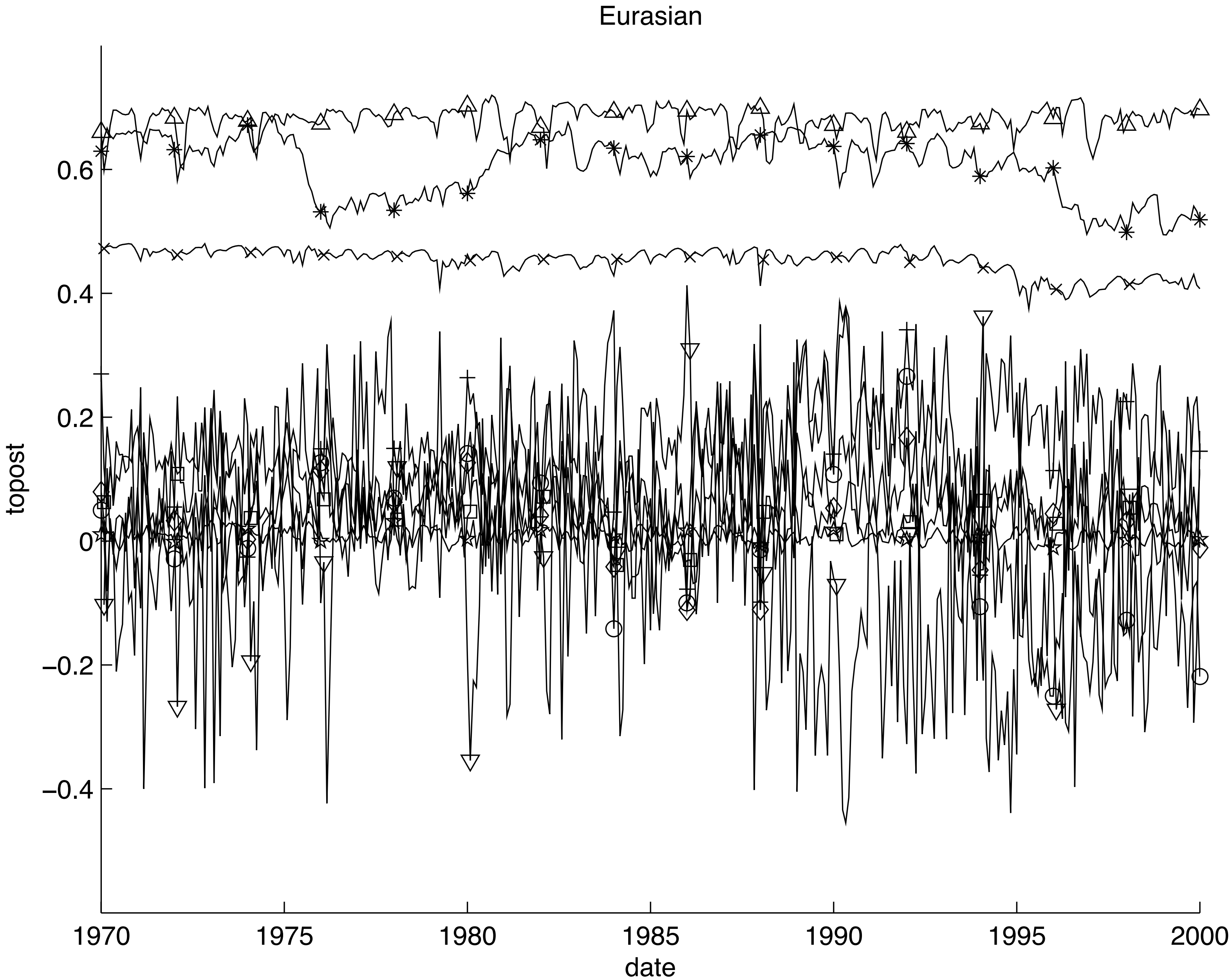
$$-1 \le \tau \le +1$$



τ

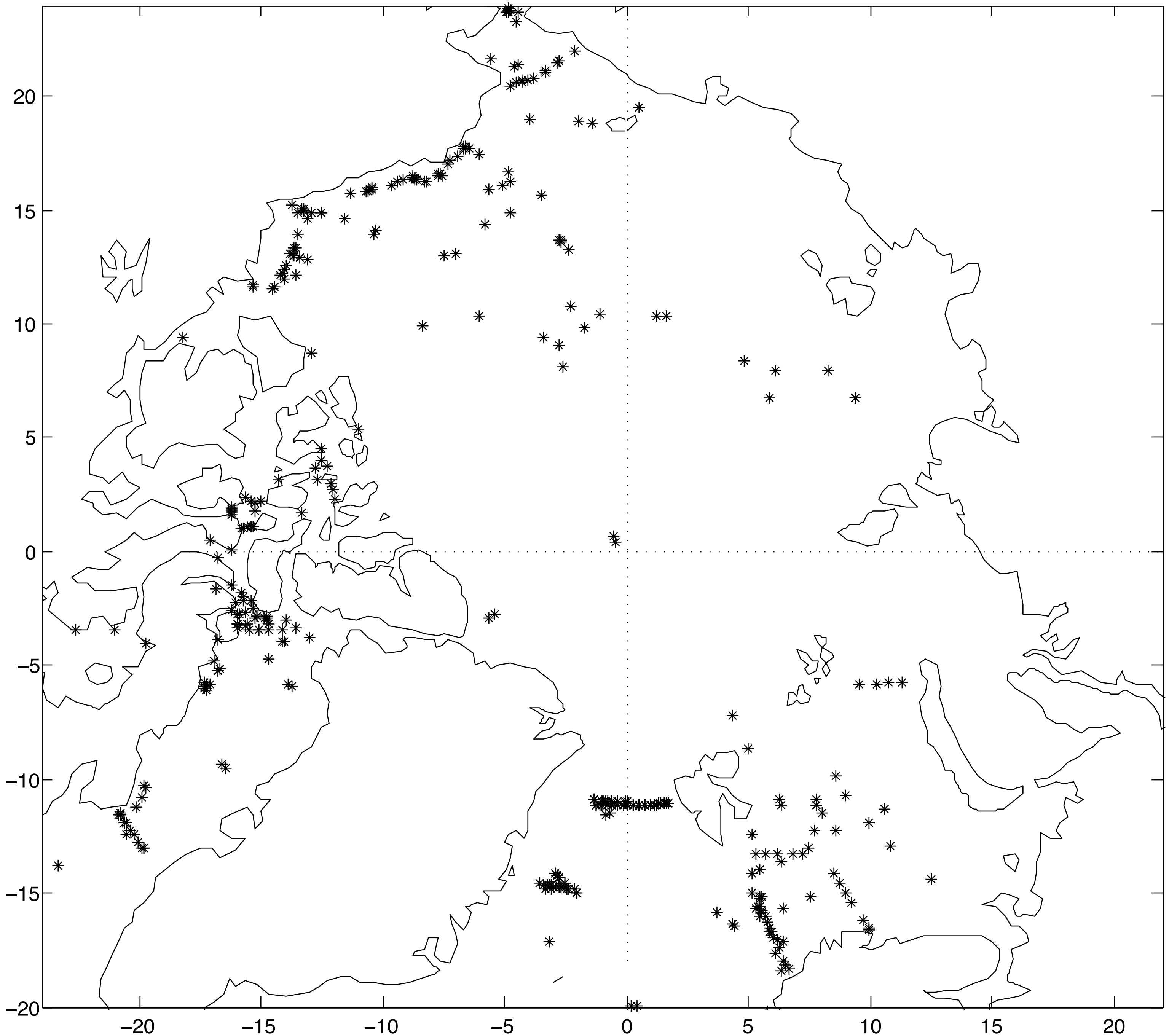

Siberia
Laptev Sea
Kara Sea
Barents Sea
Bering Strait
Chukchi Sea
Alaska
Beaufort Sea
Canada
Greenland
Fram Strait
Greenland Sea
3000
200
1000
2000
3000
1000
200
3000

Topostrophy averaged over Eurasian basin

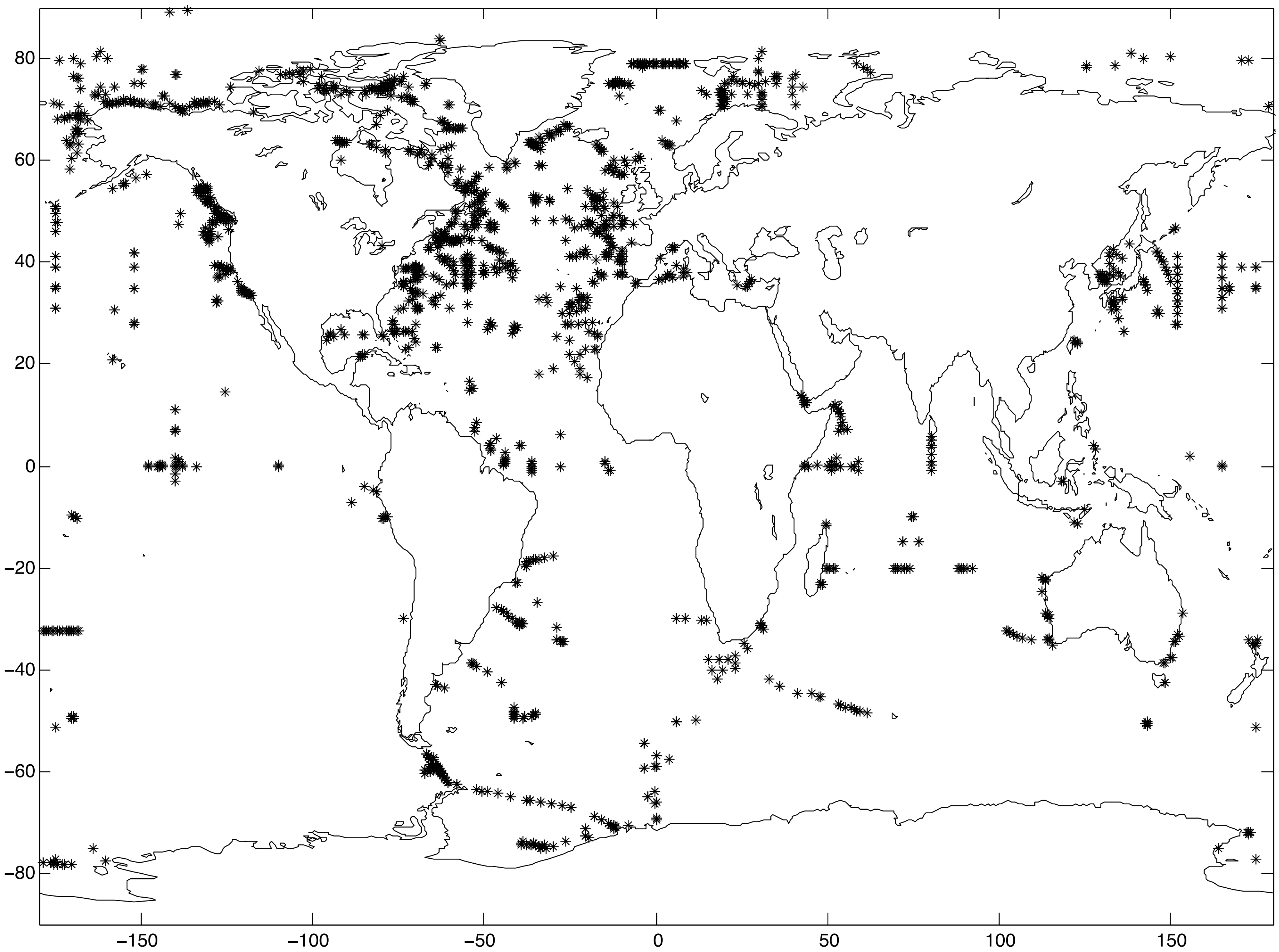


Interesting, but what is observed?

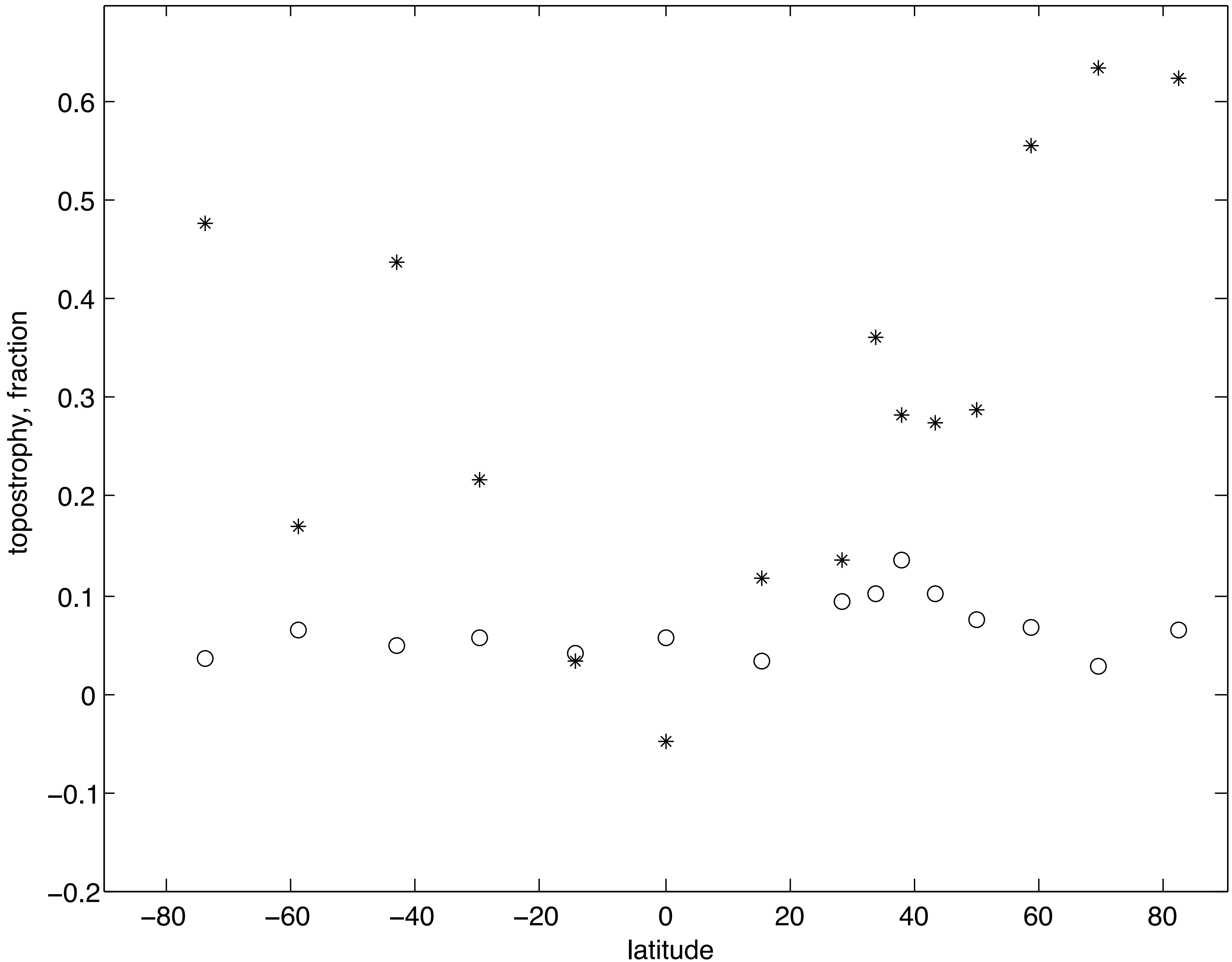
Can we estimate topostrophy from current meter records?



17120 CM records, 83087 months later ...



vs latitude nonuniform bins, here omitting top 500m



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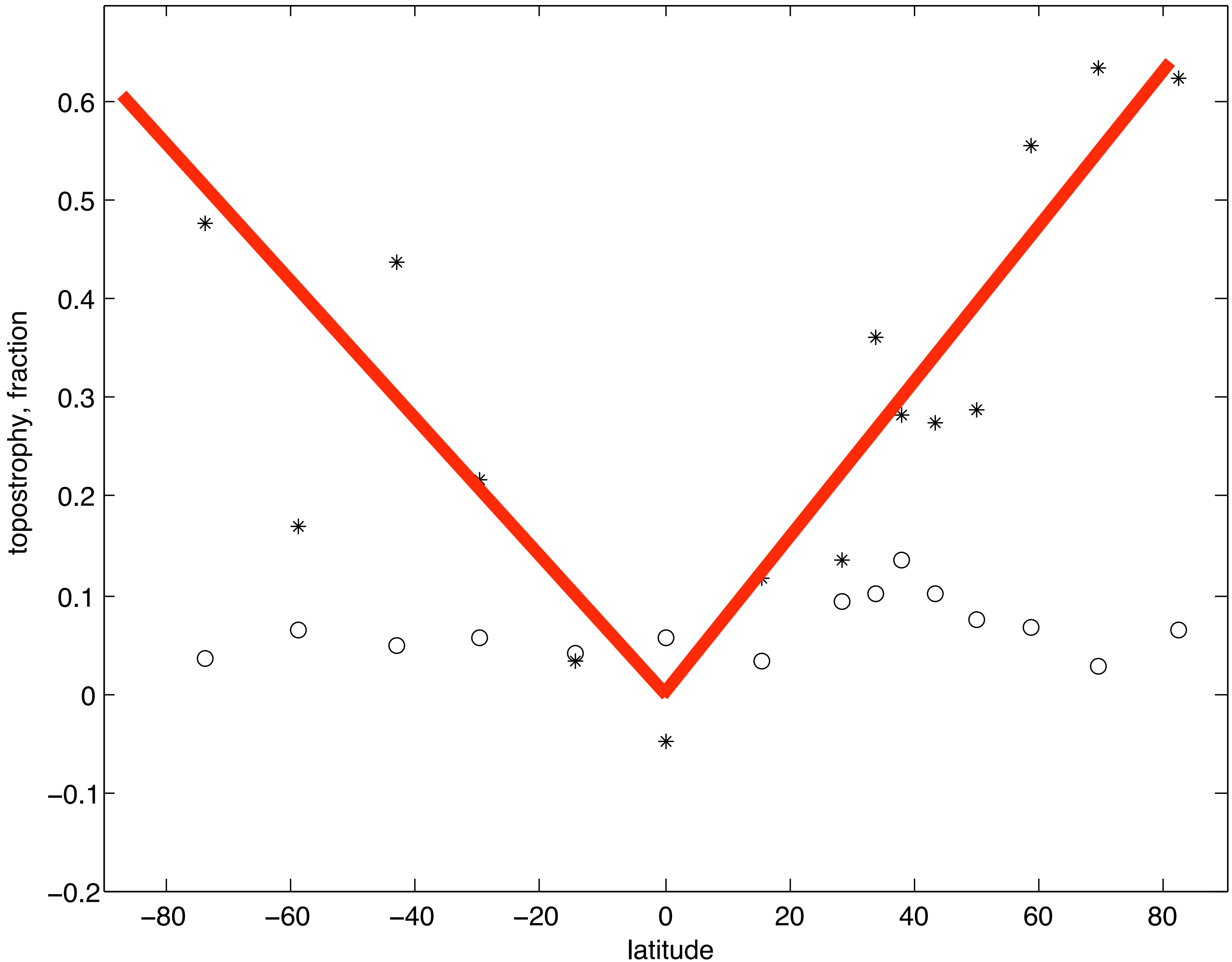
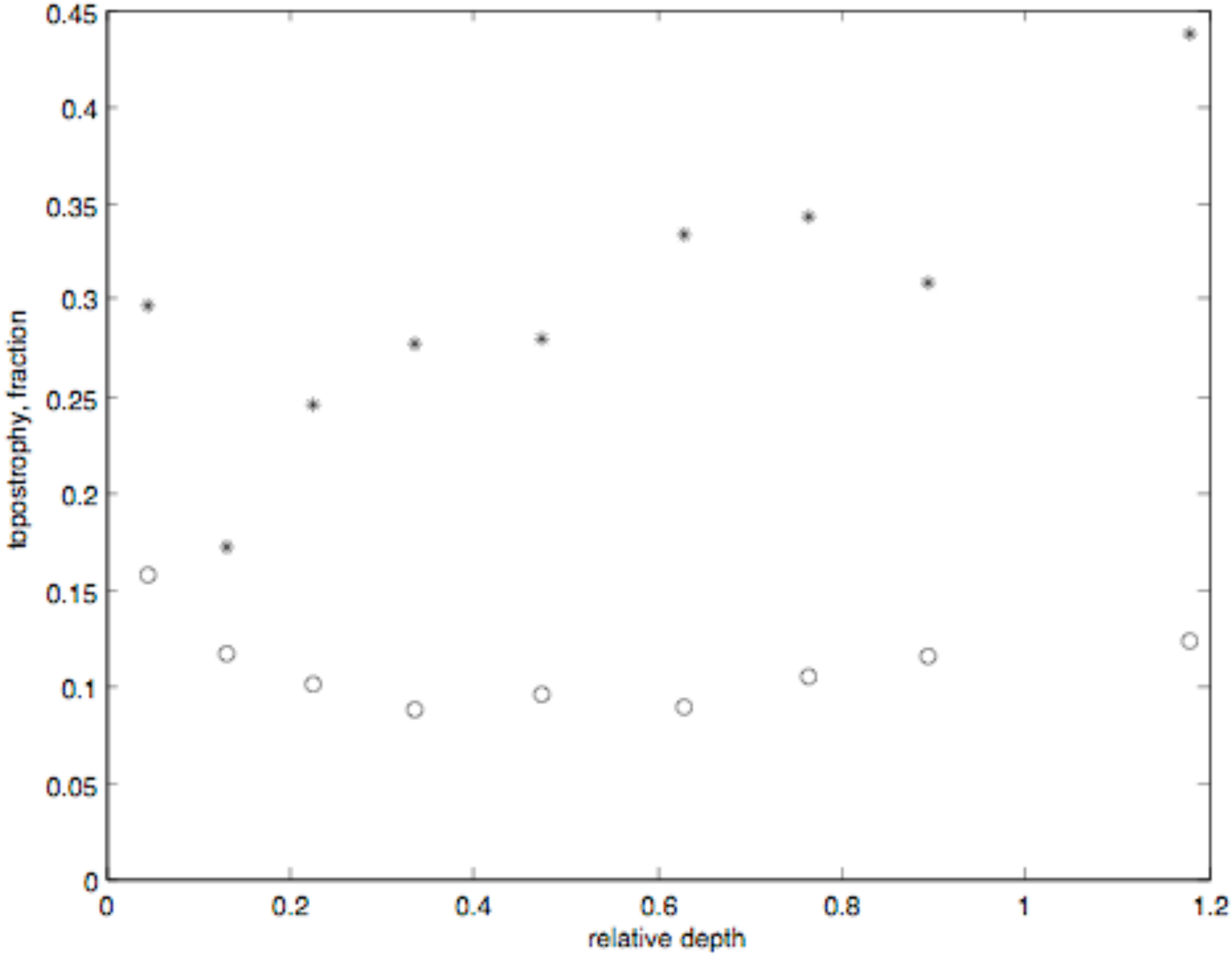
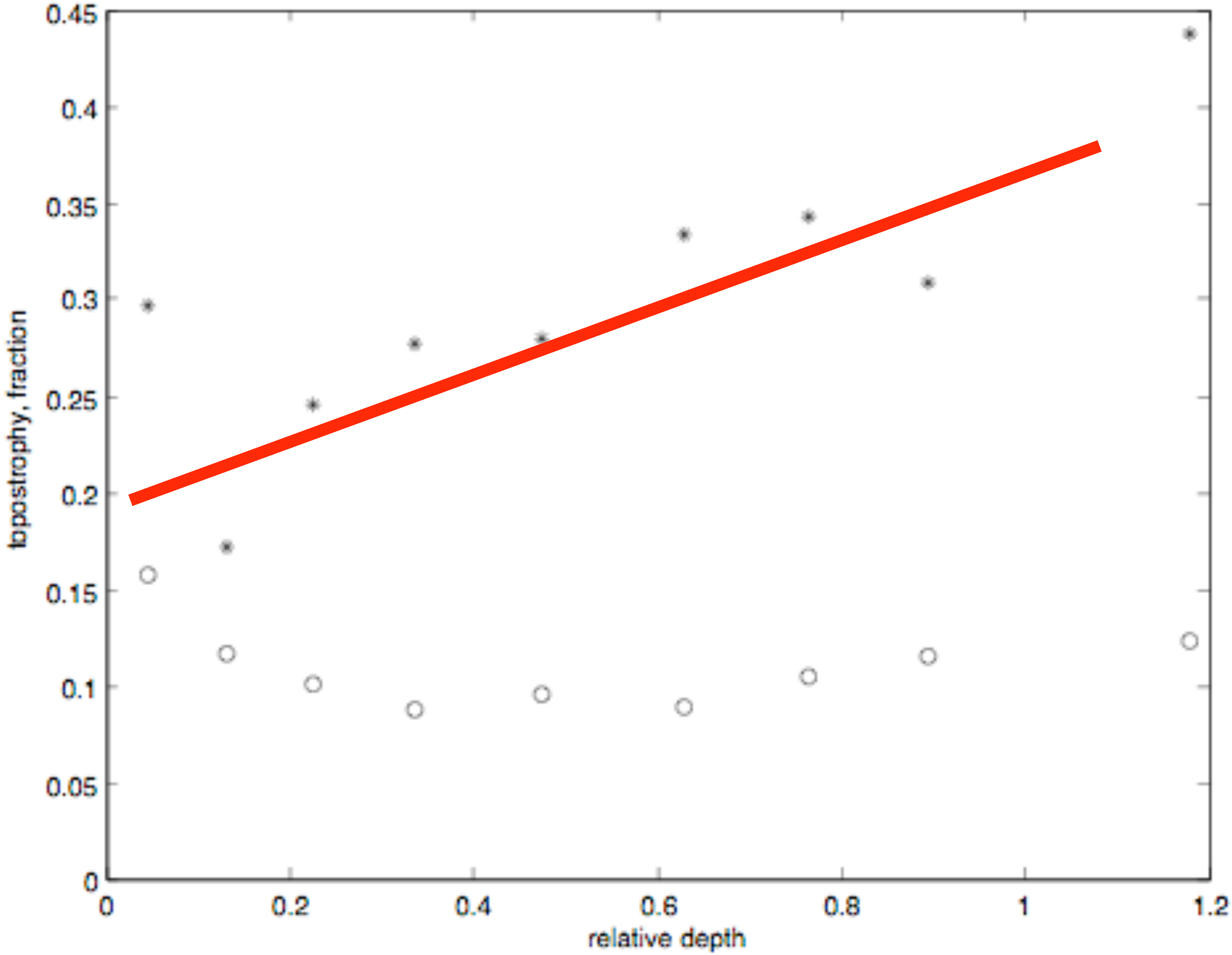


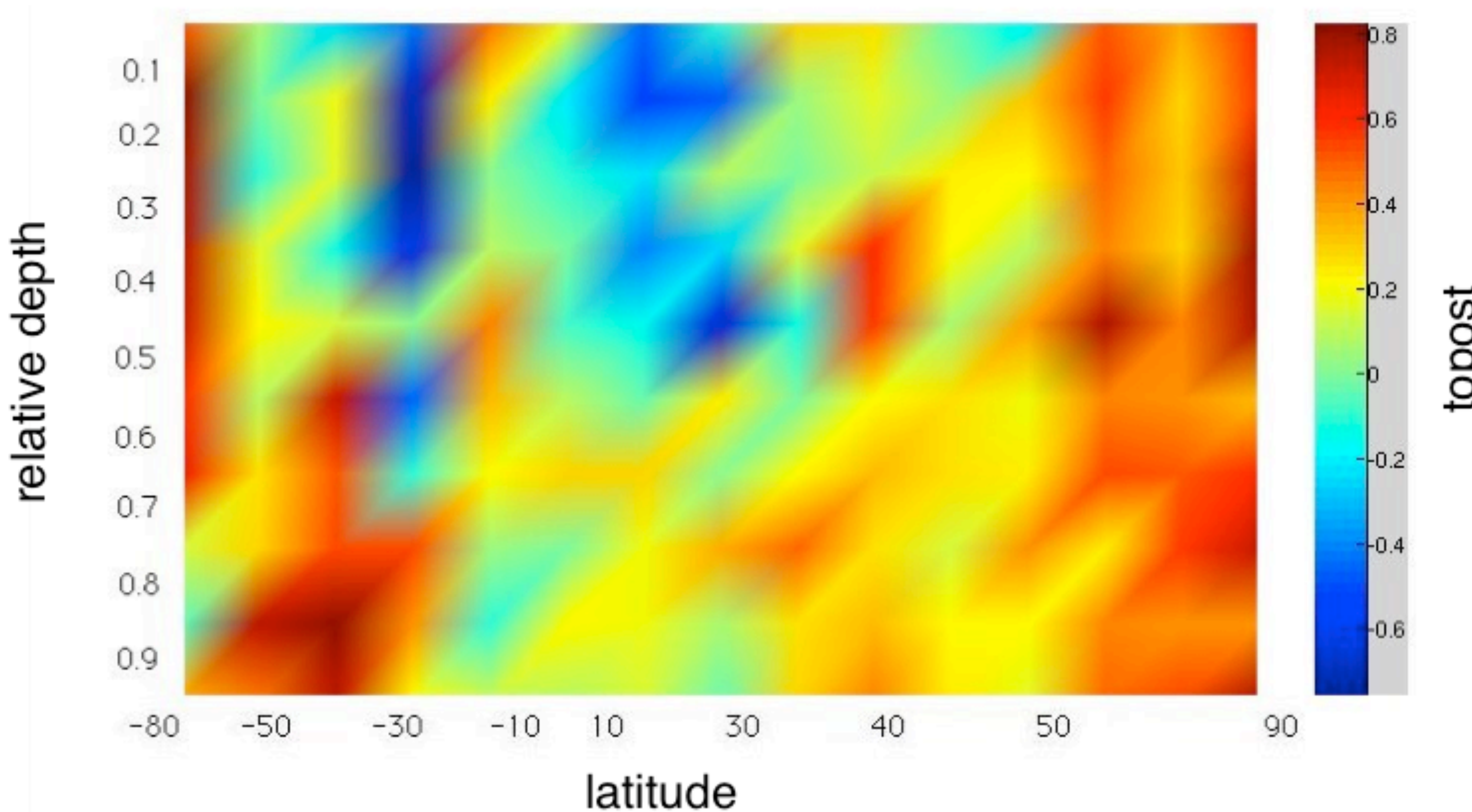
All latitudes, vs. relative depth



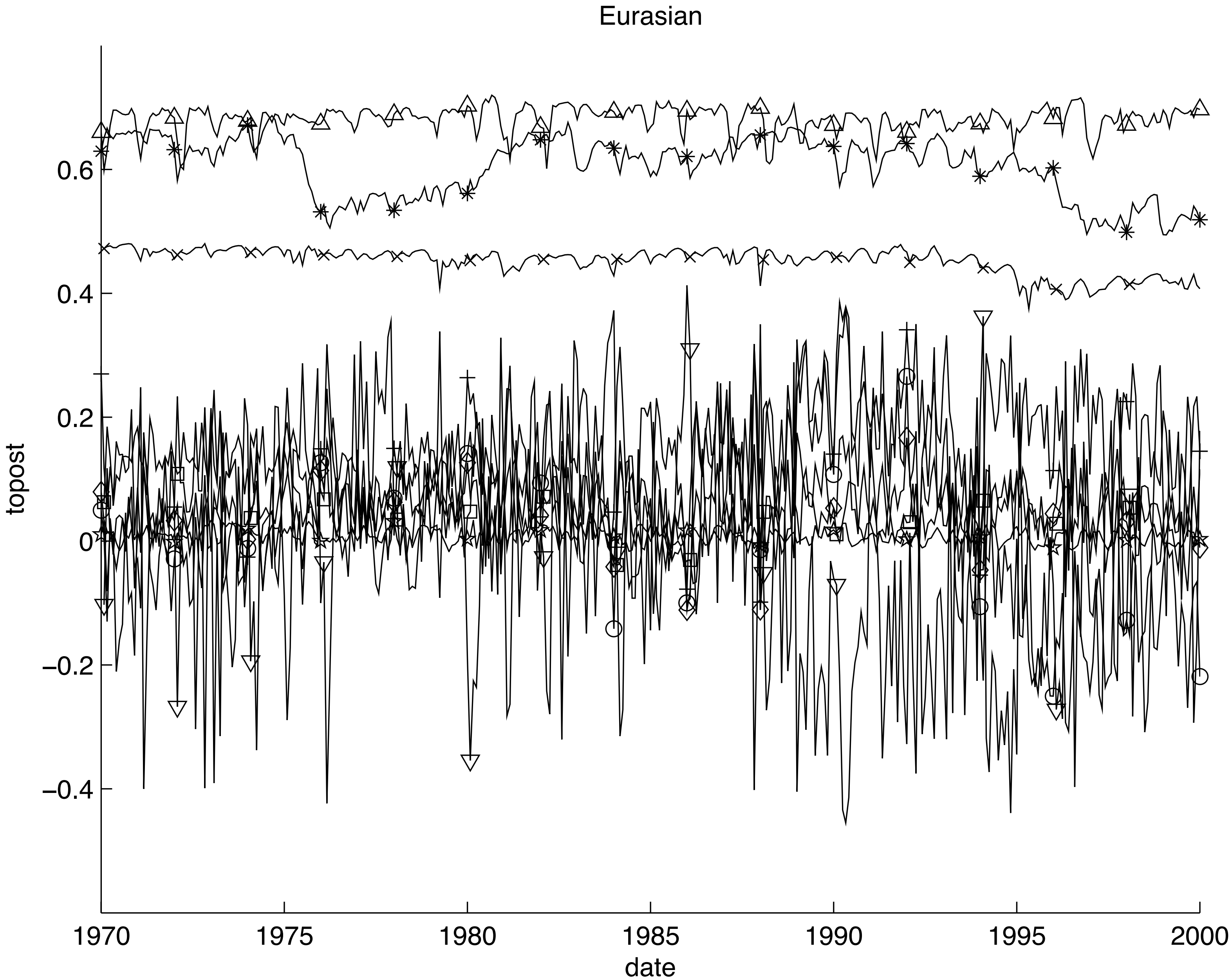
All latitudes, vs. relative depth



Topostrophy vs. latitude and relative depth



Topostrophy averaged over Eurasian basin



Problem: oceans, lakes (duck ponds?) are too big.

See 10^{24} to 10^{30} excited degrees of freedom. Get a bigger computer? Even biggees carry state vectors of maybe 10^{10}. For every variable resolved, one must guess dependence 10^{15} unknowns. Rethink!

Back to basics: what are the **equations of motion?**

Back to *even more basic*: **motion of what?**

Dependent variables as expectations:

$\mathbf{y}$=state vector (temp, salin, veloc, ...) $[\mathbf{y}]\sim 10^{30}$

for this $\mathbf{y}$ textbooks give us $d\mathbf{y}/dt=\mathbf{f}(\mathbf{y})+\mathbf{g}$

$dp=p(\mathbf{y})dy$: probability actual $\mathbf{y}'$ within dy of $\mathbf{y}$

expectations $\mathbf{Y=}\int \mathbf{y}dp$, $\mathbf{R}=\int \mathbf{r}(\mathbf{y})dp$. $[\mathbf{Y}]$ can be small

$d\mathbf{Y}/dt=\mathbf{F}(\mathbf{Y})+\mathbf{G}+$"more". "more" because $\mathbf{F}\neq\int \mathbf{f}dp$

what to do about "more"?

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what to do about "more"?

entropy $H = - \int dp \log(p)$

three choices:

a) consider **Y** that maximise H (closed system)

b) consider "more" that maximise production of H

c) "entropic force": $d\mathbf{Y}/dt=\mathbf{F}(\mathbf{Y})+\mathbf{G}+\mathbf{C}\cdot\partial_{\mathbf{Y}}H$

$\mathbf{C}\cdot\partial_{\mathbf{Y}}H$ has two parts: **C** and $\partial_{\mathbf{Y}}H$. *n.b*: "accessible"

$\mathbf{C}\cdot\partial_{\mathbf{Y}}H \sim \mathbf{C}\cdot\partial_{\mathbf{Y}}\partial_{\mathbf{Y}}H\cdot(\mathbf{Y}-\mathbf{Y}^*)=\mathbf{K}\cdot(\mathbf{Y}-\mathbf{Y}^*)$ where

$\mathbf{Y}^*$ only needs be evaluated at "small" $\partial_{\mathbf{Y}}H$

... *and* you still need **K**

In plain words --

1) entropy ($-\int \log(p)\,dp$) is "starved" at short scales

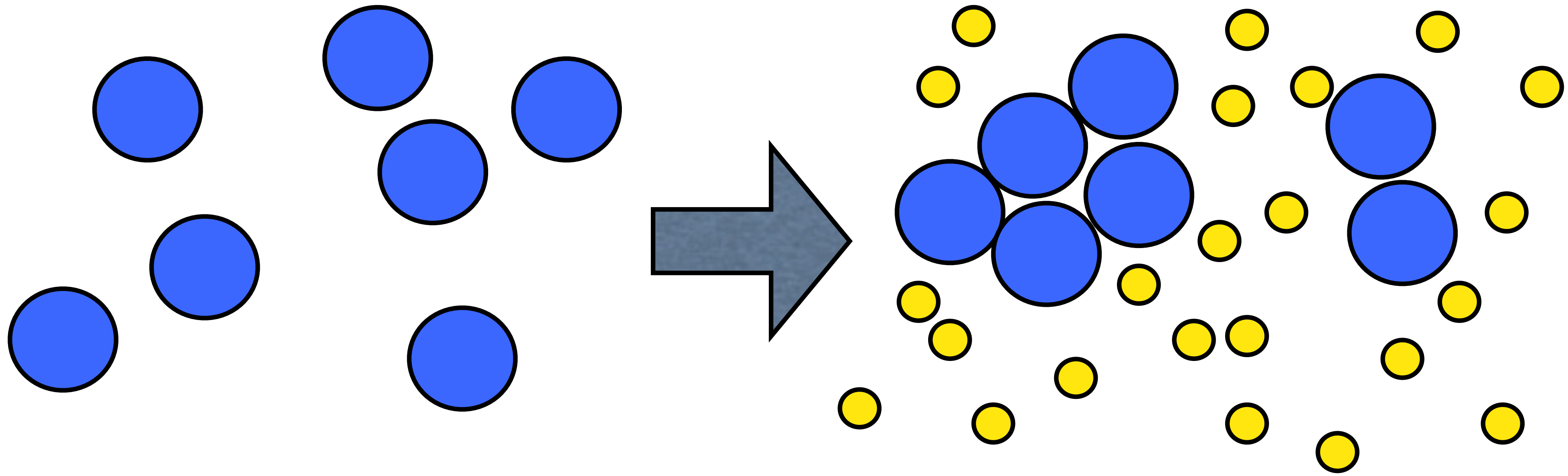
2) simplest enstrophy $(\zeta + h)^2 = \zeta^2 + 2\zeta h + h^2$

3) organizing a little $\zeta h < 0$ (losing entropy)

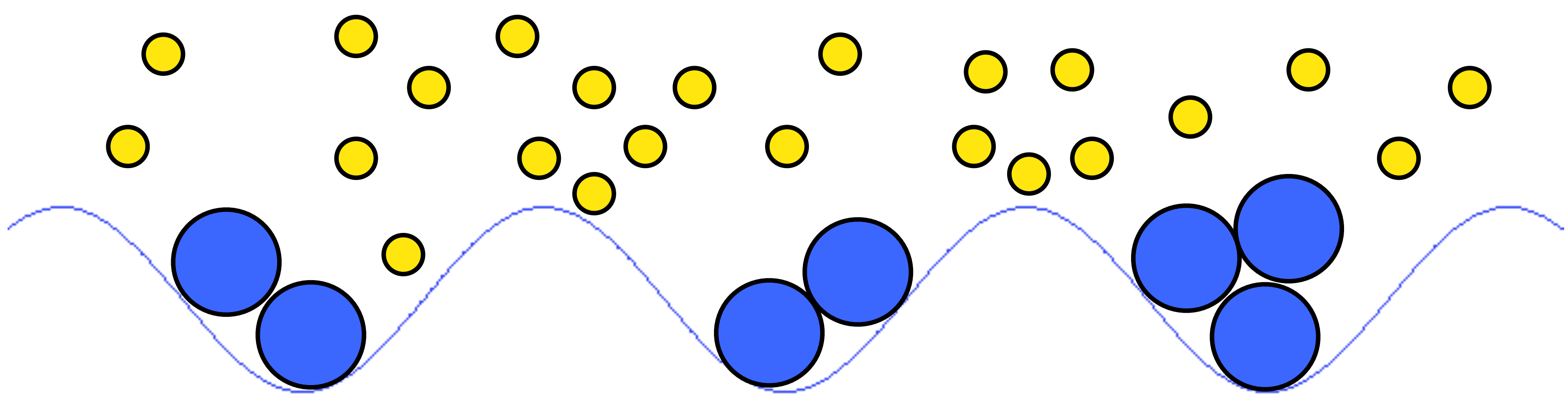
4) generates ζ^2 (=short scales, gaining entropy)

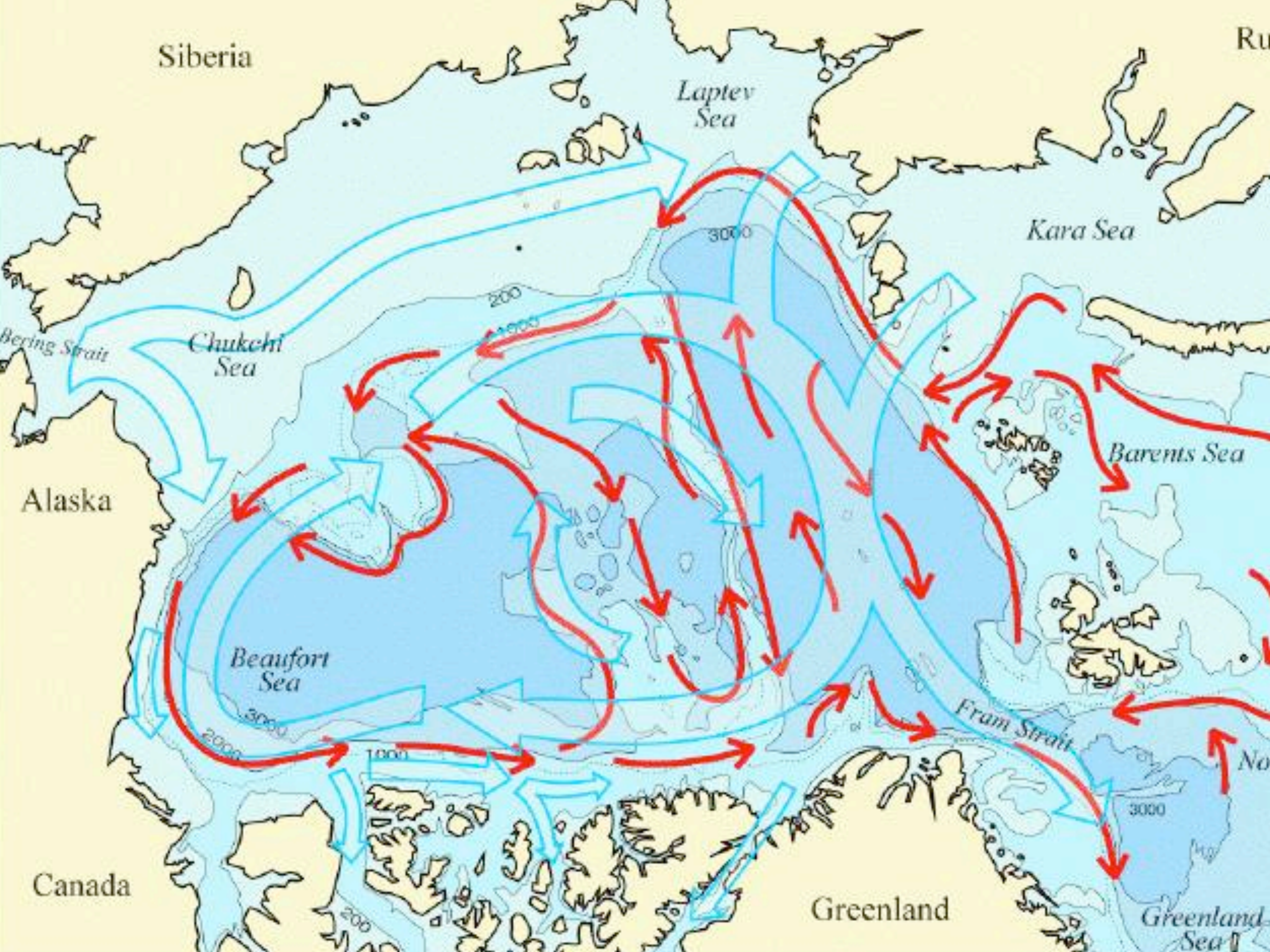
5) hence "entropic forcing" drives $\zeta \Rightarrow -h$

or $\mathbf{V} \Rightarrow -\mathbf{f} \times \nabla D$ or $\tau > 0$



Examples from nanoworld (colloids, ‘machines’, microbiol): The only explicit physics is repulsion among balls, and from walls. “See” attraction. “Entropic forcing” in the lab!





Siberia
Laptev Sea
Kara Sea
Barents Sea
Bering Strait
Chukchi Sea
Alaska
Beaufort Sea
Canada
Greenland
Fram Strait
Greenland Sea
3000
200
1000
2000

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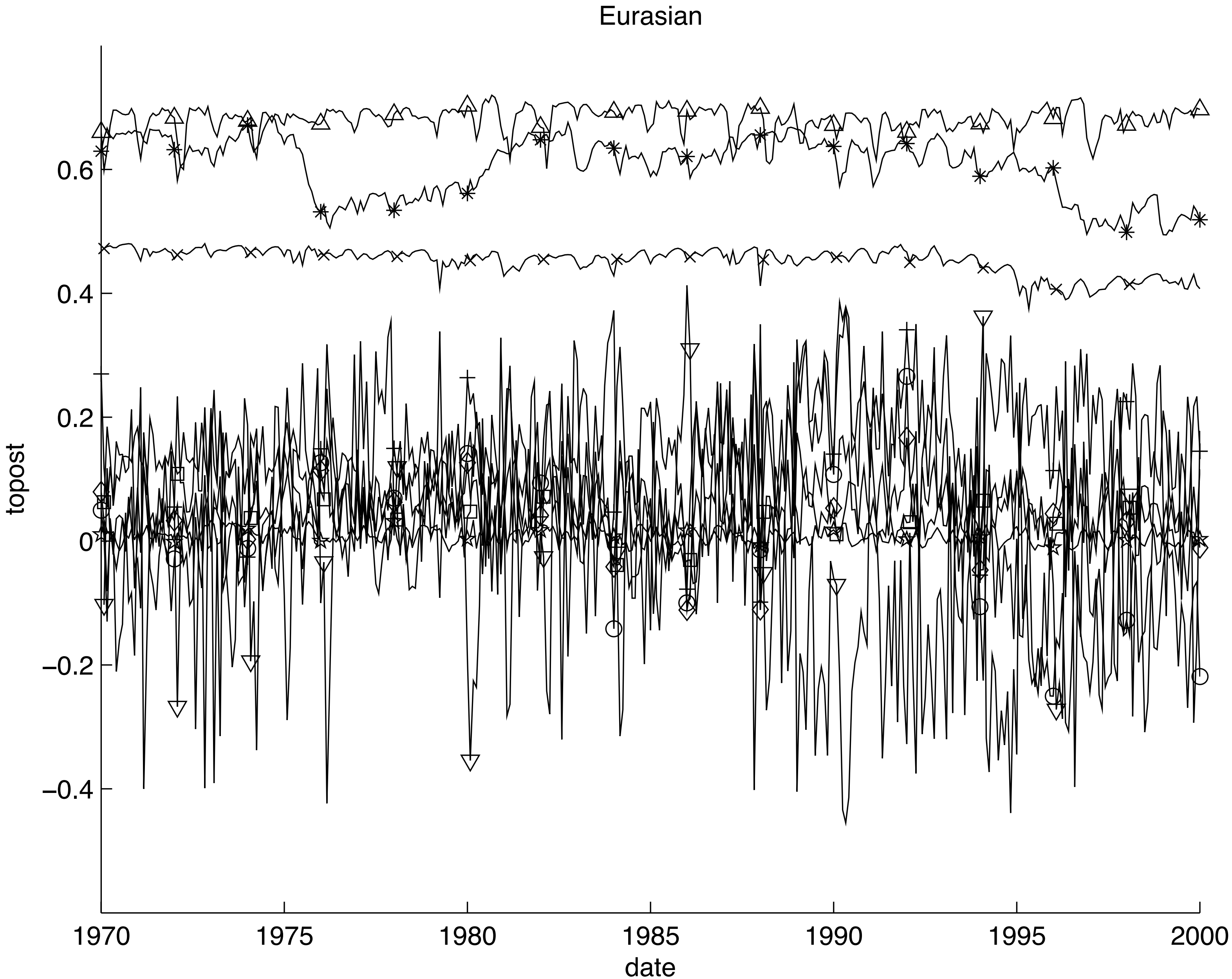
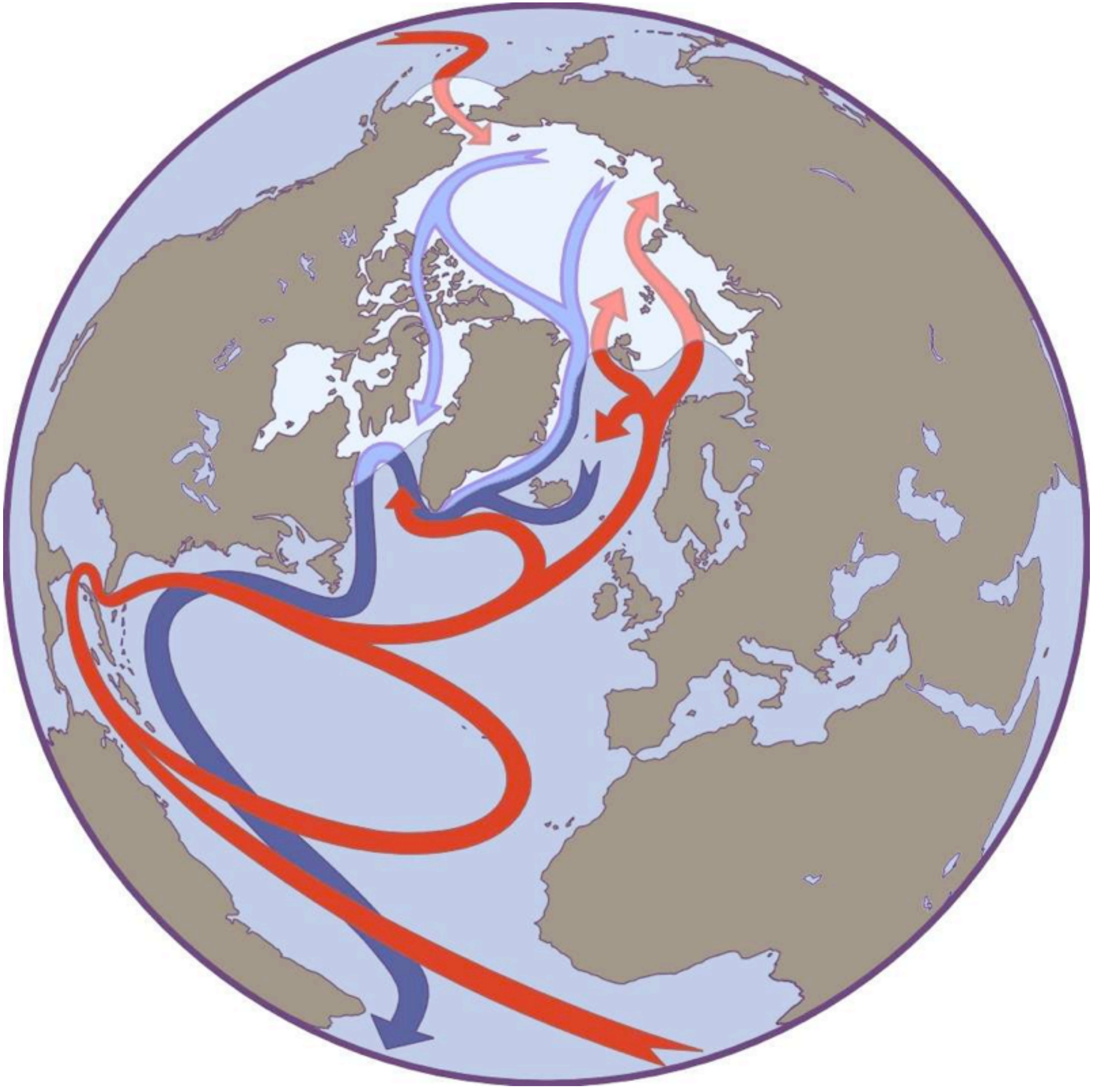


Summary

1. Topostrophy as circulation diagnostic
2. See dependent variables as expectations
3. Entropy gradients force expectations
4. *E.g*: eddy forcing mean flow along slopes

Outlook

1. Work at less fudge
2. Alternatives (max entropy production, ...?)
3. What to do about eddy-admitting models?
4. Further applications (sea ice, ...?)



Wooly words! See explicit *e.g.* $\partial_t \nabla^2 \psi + J(\psi, \nabla^2\psi + h) = \ldots$

expand $\psi = \sum \psi_n(t)\phi_n$, $h = \sum h_n \phi_n$ on eigenfunctions $\nabla^2 \phi_n + q_n^2 \phi_n = 0$

Conserved quadratics are $E = \frac{1}{2}\sum q_n^2 |\psi_n|^2$ and $\Omega = \frac{1}{2}\sum |-q_n^2\psi_n + h_n|^2$ (circulation=0 here)

Maximise $S = -\int dp \log p$ subject to $\langle E \rangle = E_0$, $\langle \Omega \rangle = \Omega_0$ and $\langle 1 \rangle = 1$.

$\delta \int \mathbf{dx}(p \log p + \alpha E p + \beta \Omega p + \gamma p) = 0$ hence $\log p + 1 + \alpha E + \beta \Omega + \gamma = 0$

$$p = \exp\{-1-\gamma\}\exp\left\{-\sum\left\{q_n^2(\alpha + \beta q_n^2)|\psi_n|^2 - 2\beta q_n^2 \operatorname{Re}\psi_n h_n + \beta |h_n|^2\right\}\right\}$$

$= \Gamma \exp\left\{-\sum q_n^2(\alpha + \beta q_n^2)|\psi_n - \hat{\psi}_n|^2\right\}$ where $\hat{\psi}_n = \beta h_n / (\alpha + \beta q_n^2)$ or $(\alpha/\beta - \nabla^2)\hat{\psi} = h$

If resolved scales are larger than $\lambda = \sqrt{\beta/\alpha}$, drop ∇^2 and simply $\hat{\psi} = \lambda^2 h$

K·(**Y**-**Y***) with **Y***=-fL^{2}D, **K**=A∇^2 : “neptune”