Parameterizing geostrophic eddies in ocean models: energetics, potential vorticity mixing and flow stability

David Marshall *(University of Oxford)*

Alistair Adcroft *(GFDL)*

Structure:

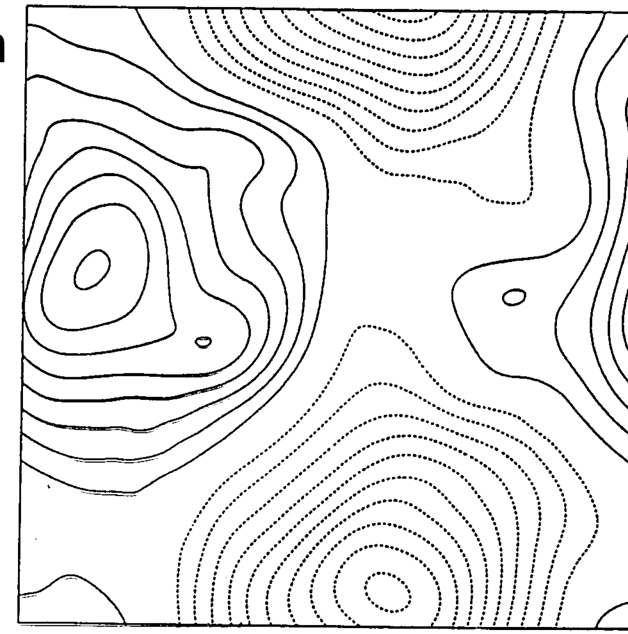
1. Barotropic theory
2. Extension to a baroclinic, quasigeostrophic ocean
3. Application to OGCMs

Geostrophic turbulence ~ 2-d turbulence

⇒ some quantities (e.g., vorticity) cascade to small scales and are mixed; other quantities (e.g., energy) cascade to large spatial scales and are quasi-conserved.

vorticity

streamfunction



(calculation: Vallis and Maltrud)

⇒ *constraints* imposed on geostrophic eddy fluxes

moreover, can solve explicit EKE budget to constrain eddy transfer coefficients

1. Theory for barotropic model

momentum equation:

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{k} \times q\mathbf{u} + \nabla B = \mathbf{F} - \mathbf{k} \times \overline{q'\mathbf{u}'} - \nabla \frac{\overline{\mathbf{u}' \cdot \mathbf{u}'}}{2}$$

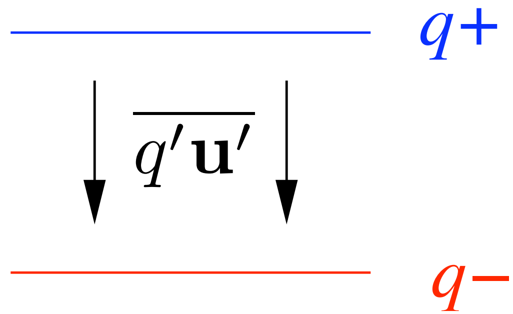
can decompose into rotational and divergent components:

only eddy force that can drive a net acceleration

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{k} \times (q\mathbf{u})_{\mathrm{div}} \qquad = \quad \mathbf{F}_{\mathrm{rot}} - \mathbf{k} \times \boxed{\overline{q'\mathbf{u}'}_{\mathrm{div}}},$$

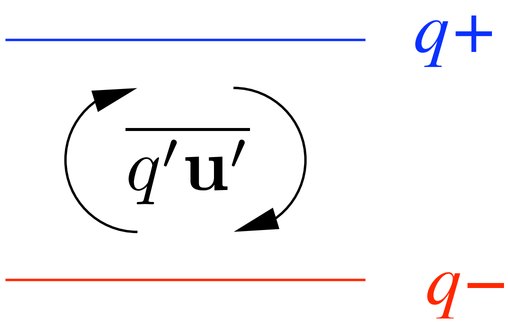
Hoskins (1983), Hughes and Ash (2001), Marshall and Shutts (1981), ...

(a) vorticity mixing



$\Rightarrow$ eddy vorticity flux down-gradient

$$\overline{q'\mathbf{u}'} = -\kappa \nabla q + \mathbf{k} \times \nabla \lambda$$



cf. Green (1970)

(b) energy conservation

Can derive, without further approximation:

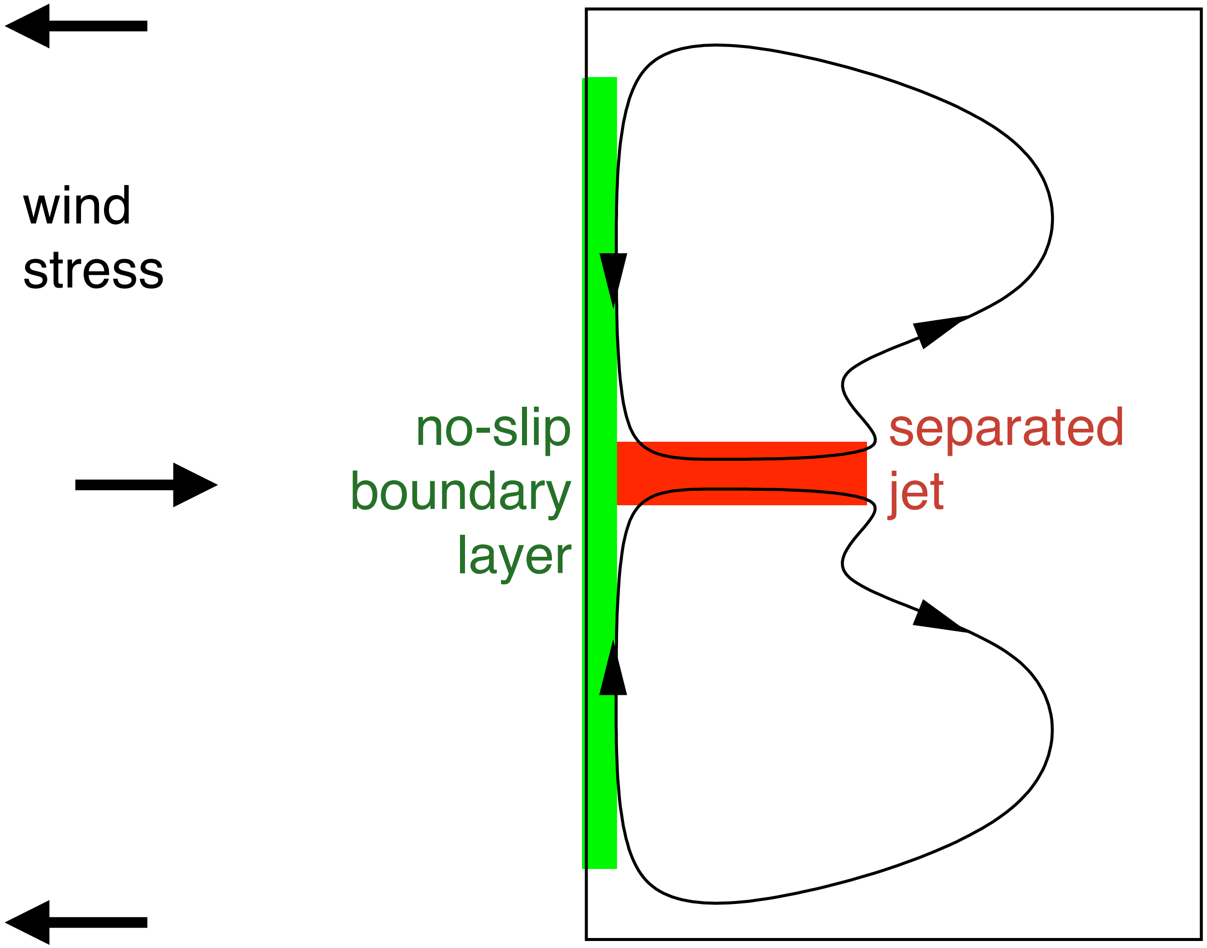
$$\frac{\partial}{\partial t}(\text{eddy energy}) = \boxed{-\kappa\, \mathbf{u}\cdot\mathbf{u}\, \frac{\partial q}{\partial \psi}_{\perp}} + \nabla \cdot (\text{flux})$$

mean to eddy energy conversion

$$\kappa = \gamma U_{eddy} l_{eddy} = \gamma l_{eddy}\sqrt{2\ (\text{eddy energy})}$$

also include diffusion and Newtonian damping of eddy energy (+ ...)

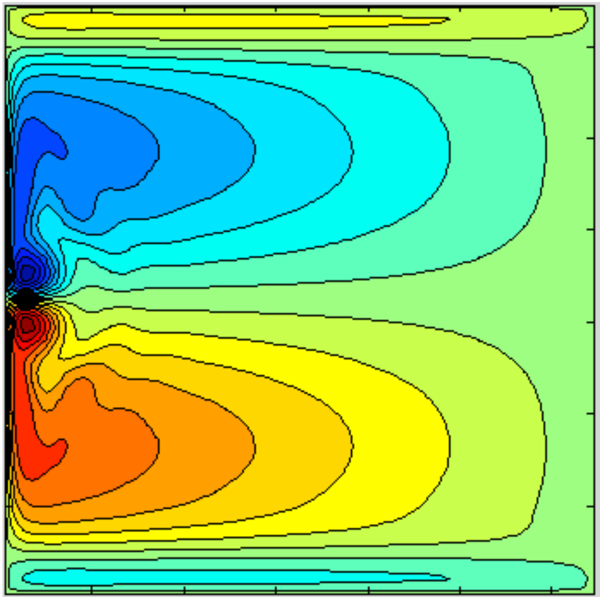
Regions where $\partial q / \partial \psi < 0 \Rightarrow$ eddy growth:



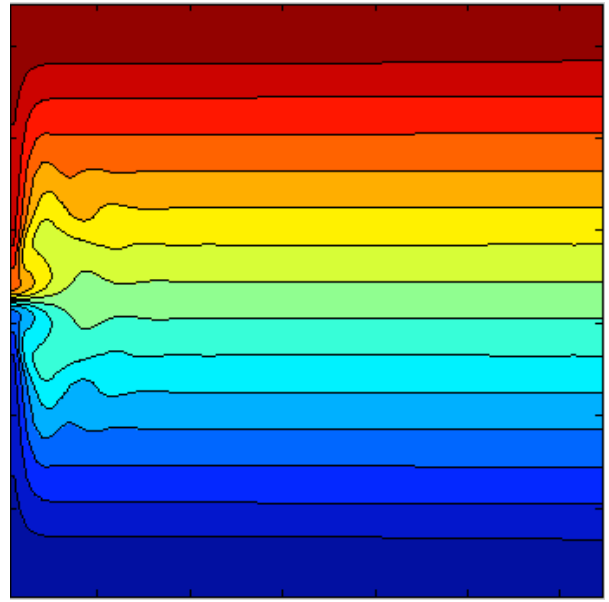
Wind-driven gyres (free-slip):

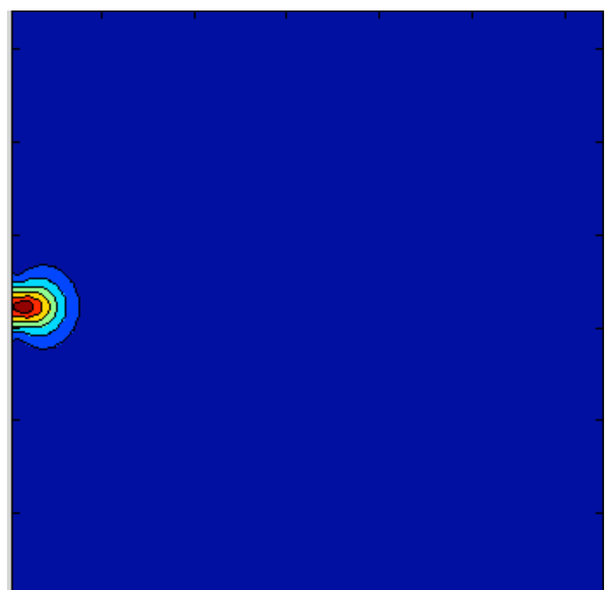
wind
stress

streamfunction



vorticity

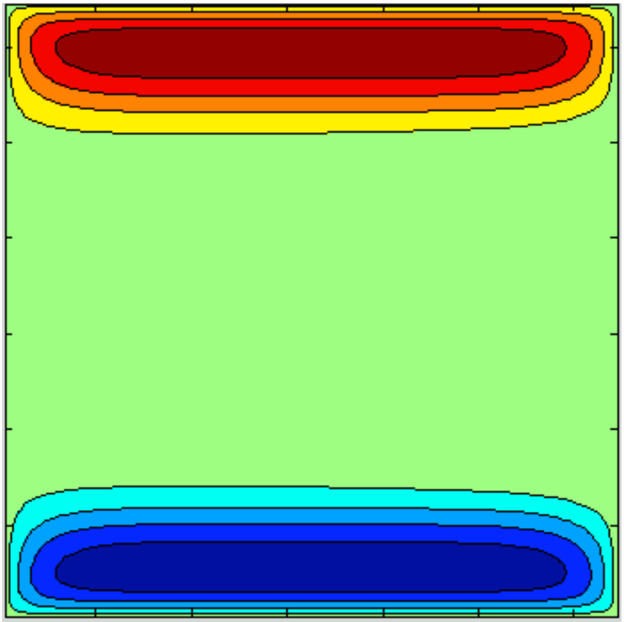




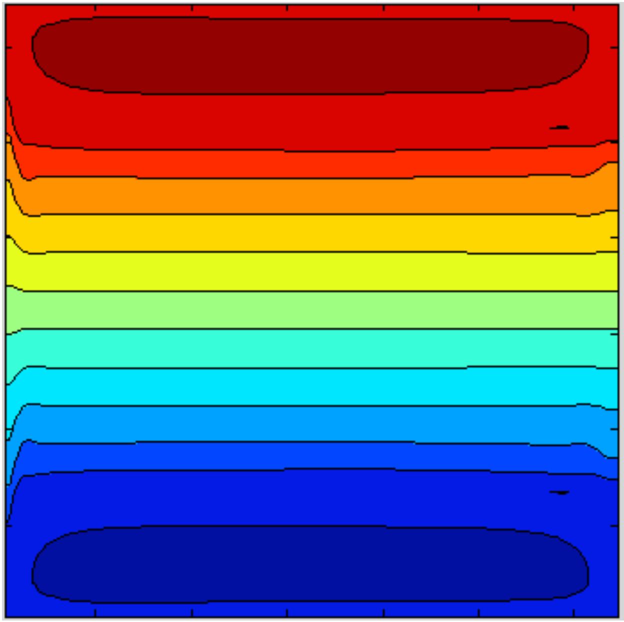
eddy kinetic energy

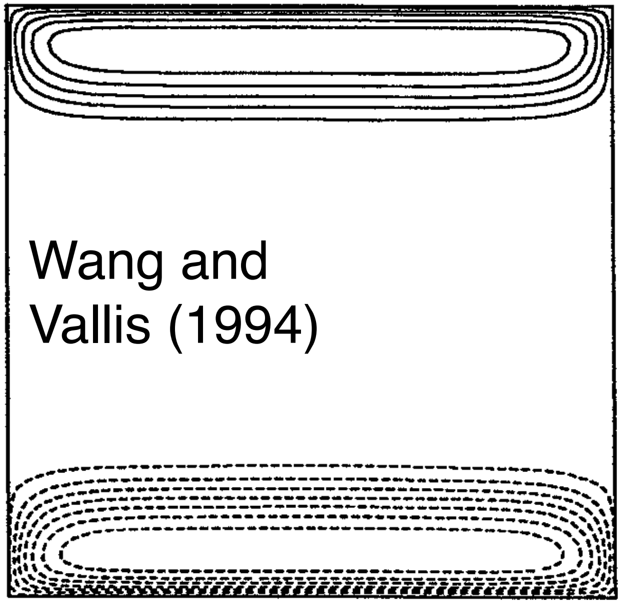
Parameterized freely-decaying turbulence (initial uniform eddy energy):

streamfunction



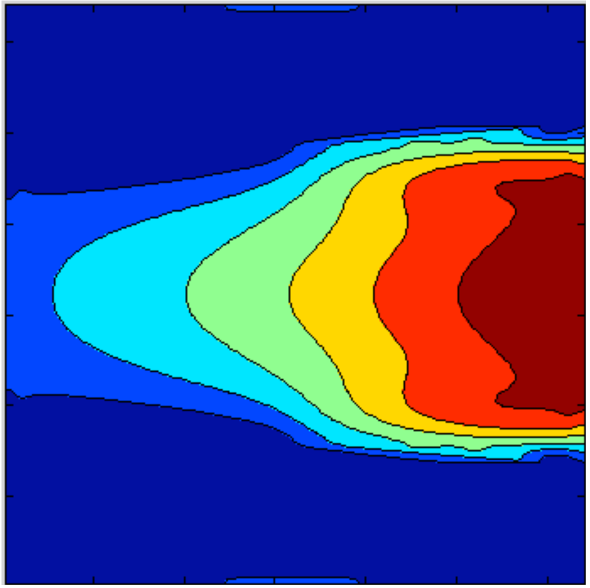
vorticity





"Fofonoff gyres"
($\partial q/\partial\psi$ > 0)

eddy
kinetic
energy



2. Theory for a layered quasigeostrophic model

Assume down-gradient closure for *potential vorticity*
(plus arbitrary rotational gauge):

$$\overline{Q'\mathbf{u}'} = -\kappa \nabla Q + \mathbf{k} \times \nabla \lambda$$

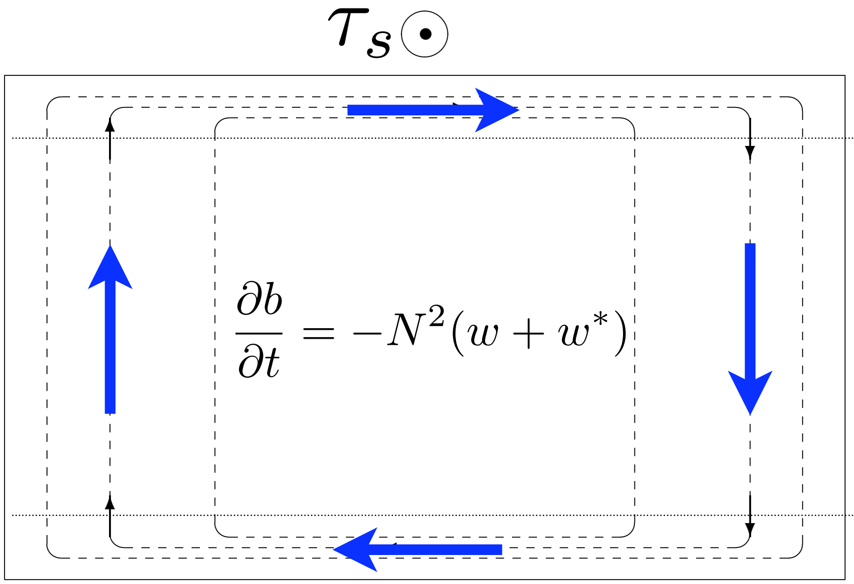
Then: $$\frac{\partial}{\partial t}(EKE + EPE) = -\sum_{i=1}^{N} H_i \kappa_i \frac{\partial Q_i}{\partial \psi_i}_{\perp} \mathbf{u}_{gi} \cdot \mathbf{u}_{gi} + \cdots$$

Thus, need $\frac{\partial Q_i}{\partial \psi_i}_{\perp} < 0$ for eddy energy to grow

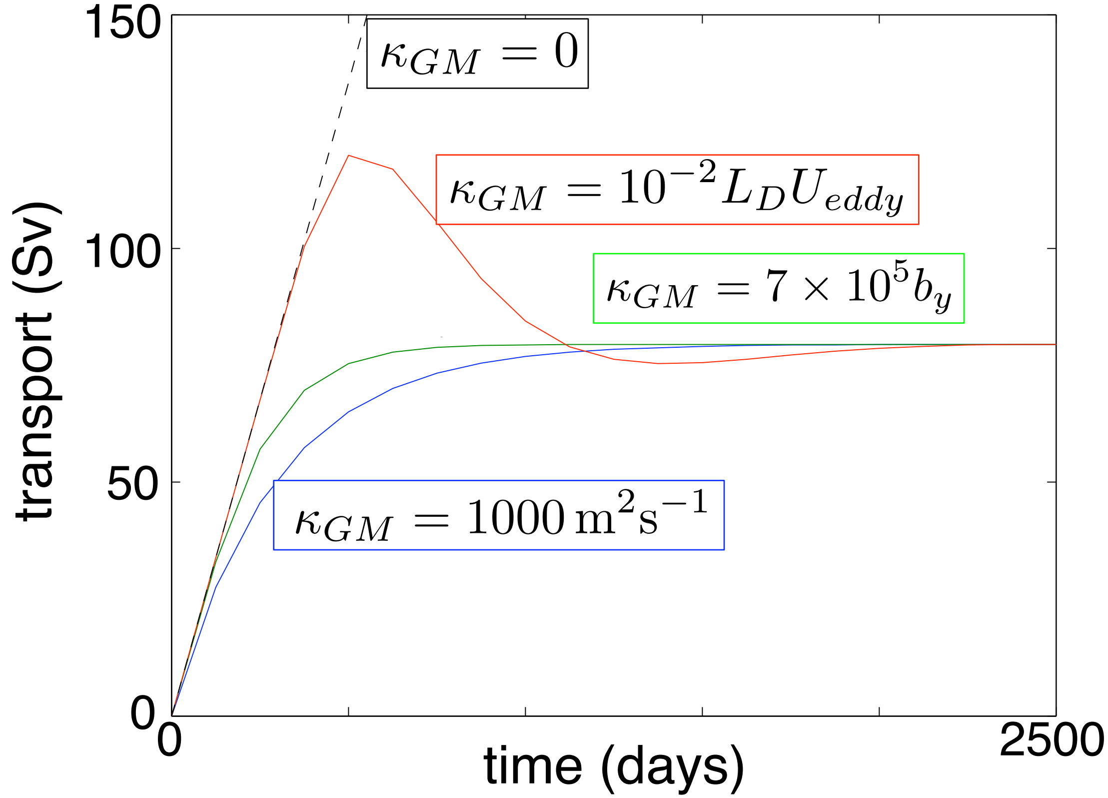
(Arnold, Charney-Stern, ...)

For now, use isopycnal layer thickness following Gent and McWilliams (1990)

Spin up of thermal wind transport in a wind-driven zonal channel:

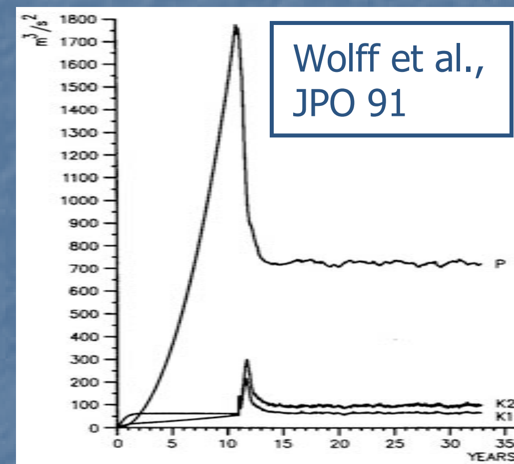


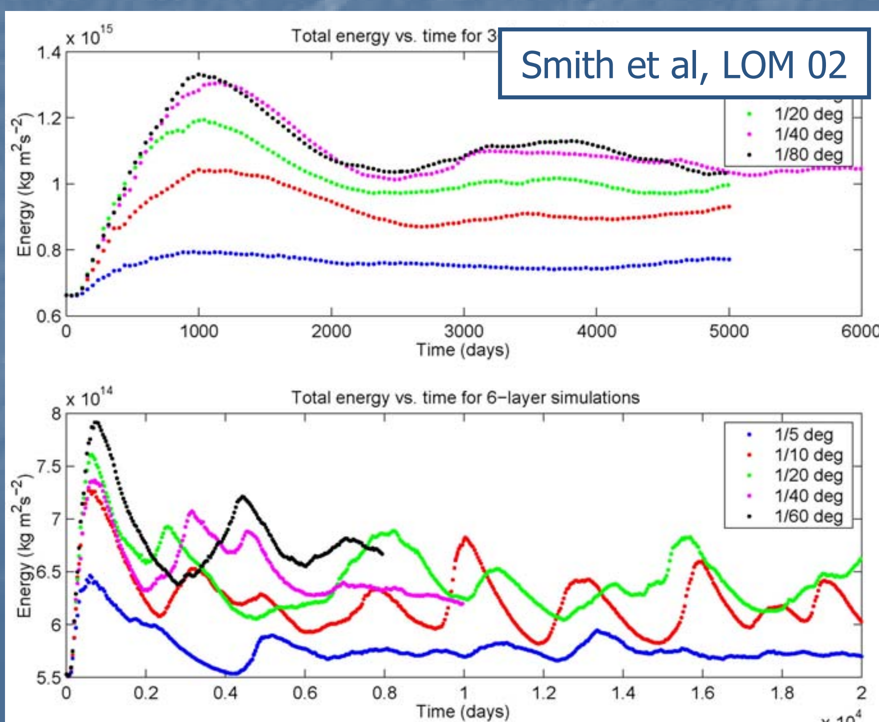
$$\boxed{\partial_t \mathcal{E} = \kappa_{GM} S^2 N^2 - \lambda \mathcal{E}}$$

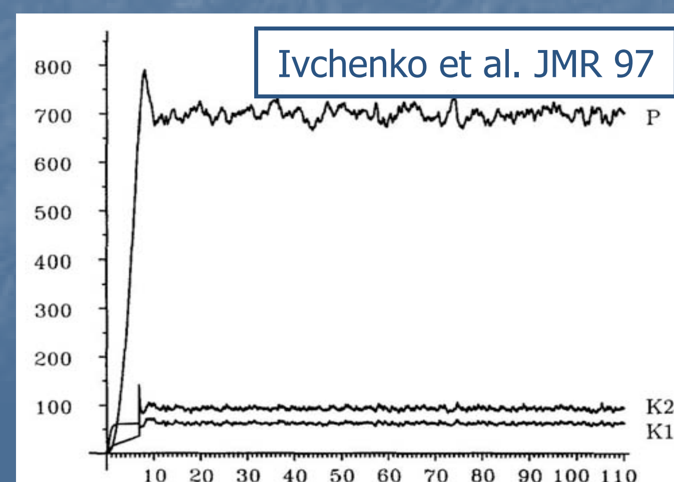


Delay in eddy activity

- Many simulations exhibit a delay in the "action" of eddies, as EKE production comes into balance with forcing







3. Application to OGCMs

Resolved kinetic energy equation

$$\boxed{\partial_t \frac{1}{2}|\underline{\overline{\mathbf{v}}}|^2 + \underline{\nabla}\cdot\overline{B}\,\underline{\overline{\mathbf{v}}} - \overline{w}\overline{b} = -\underline{\overline{\mathbf{v}}}\cdot\mathcal{P}_\zeta + \underline{\overline{\mathbf{v}}}\cdot\underline{\nabla}\cdot\underline{\underline{\overline{\tau}}}}$$

Resolved potential energy equation

$$\boxed{\partial_t(-z\overline{b}) + \underline{\nabla}\cdot(-z\overline{b}\,\underline{\overline{\mathbf{v}}}) + \overline{w}\overline{b} = \underline{\nabla}\cdot z\left(\mathcal{P}_b\right) - \underline{\hat{\mathbf{k}}}\cdot\mathcal{P}_b}$$

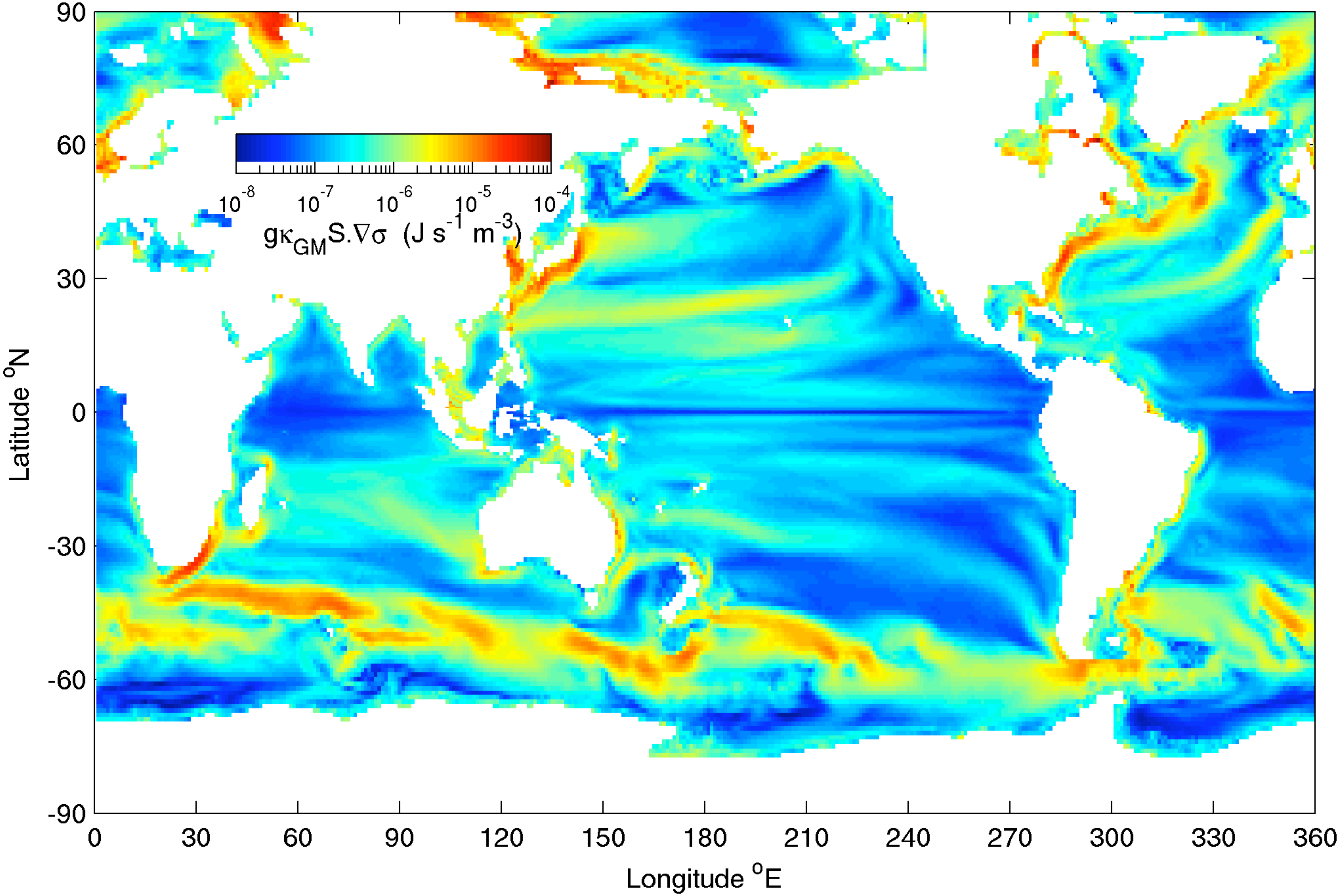
$\underline{\overline{\mathbf{v}}}\cdot\mathcal{P}_\zeta$

$\underline{\hat{\mathbf{k}}}\cdot\mathcal{P}_b$

$$\boxed{\partial_t\mathcal{E} + \underline{\nabla}\cdot\overline{B'\underline{\mathbf{v}}'} = \underline{\overline{\mathbf{v}}}\cdot\mathcal{P}_\zeta + \underline{\hat{\mathbf{k}}}\cdot\mathcal{P}_b + \overline{\underline{\mathbf{v}}'\cdot\underline{\nabla}\cdot\underline{\underline{\tau}}'}}$$

Eddy kinetic energy equation

Removal of resolved potential energy by Gent and McWilliams in GFDL coupled model:



Resolved kinetic energy equation

$$\boxed{\partial_t \tfrac{1}{2}|\overline{\underline{\mathbf{v}}}|^2 + \underline{\nabla}\cdot\overline{B}\,\overline{\underline{\mathbf{v}}} - \overline{w}\overline{b} = -\overline{\underline{\mathbf{v}}}\cdot\mathcal{P}_\zeta + \overline{\underline{\mathbf{v}}}\cdot\underline{\nabla}\cdot\overline{\underline{\underline{\tau}}}}$$

Resolved potential energy equation

$$\boxed{\partial_t(-z\overline{b}) + \underline{\nabla}\cdot(-z\overline{b}\,\overline{\underline{\mathbf{v}}}) + \overline{w}\overline{b} = \underline{\nabla}\cdot z\left(\mathcal{P}_b\right) - \hat{\underline{\mathbf{k}}}\cdot\mathcal{P}_b}$$

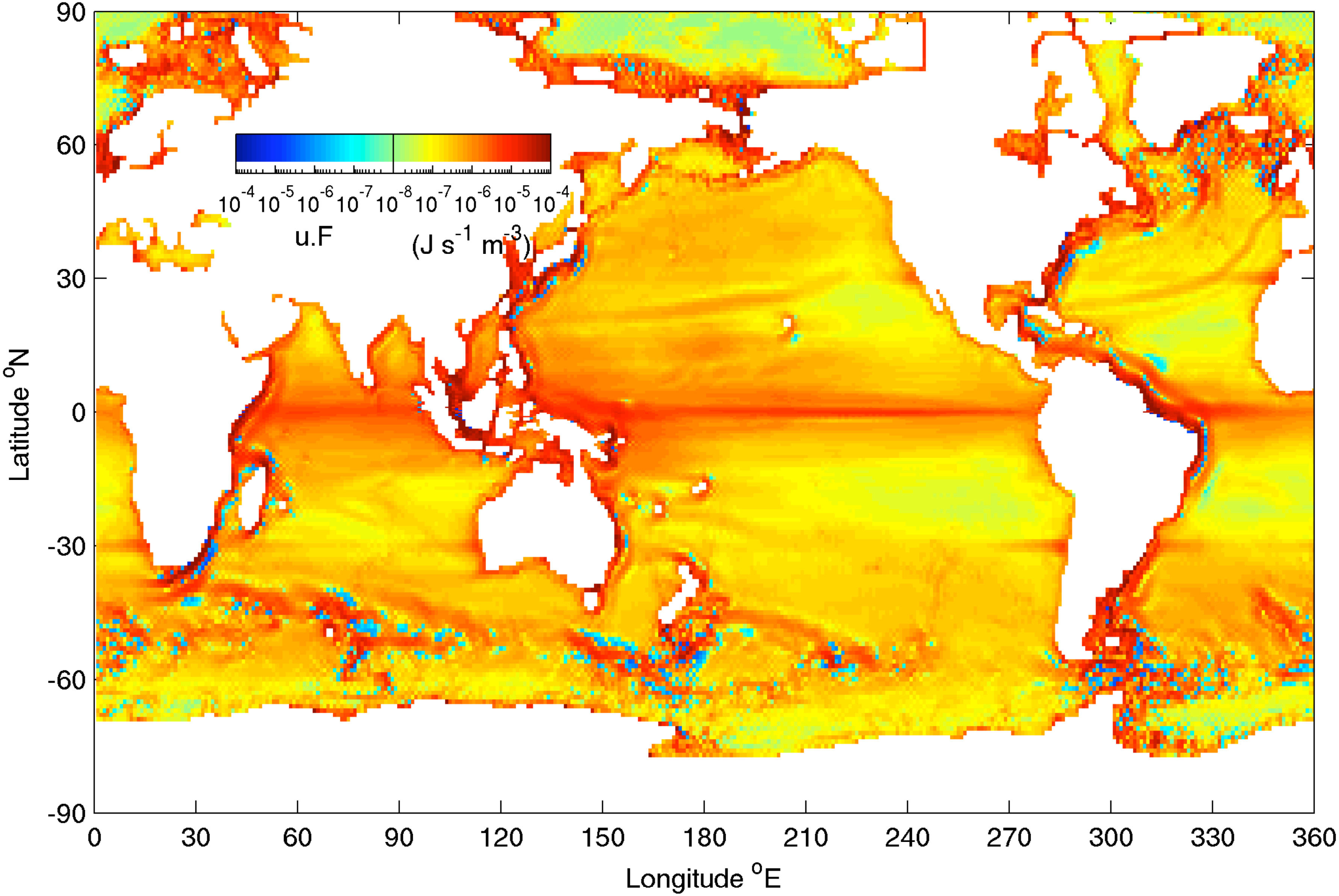
$\overline{\underline{\mathbf{v}}}\cdot\mathcal{P}_\zeta$

$\hat{\underline{\mathbf{k}}}\cdot\mathcal{P}_b$

$$\boxed{\partial_t\mathcal{E} + \underline{\nabla}\cdot\overline{B'\underline{\mathbf{v}}'} = \overline{\underline{\mathbf{v}}}\cdot\mathcal{P}_\zeta + \hat{\underline{\mathbf{k}}}\cdot\mathcal{P}_b + \overline{\underline{\mathbf{v}}'\cdot\underline{\nabla}\cdot\underline{\underline{\tau}}'}}$$

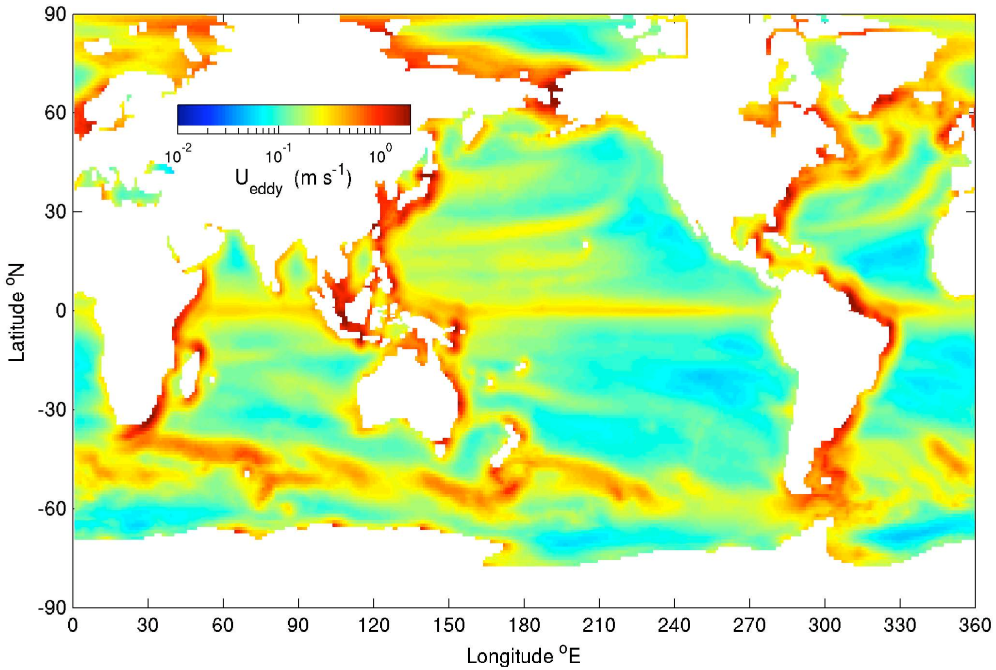
Eddy kinetic energy equation

Removal of resolved kinetic energy:

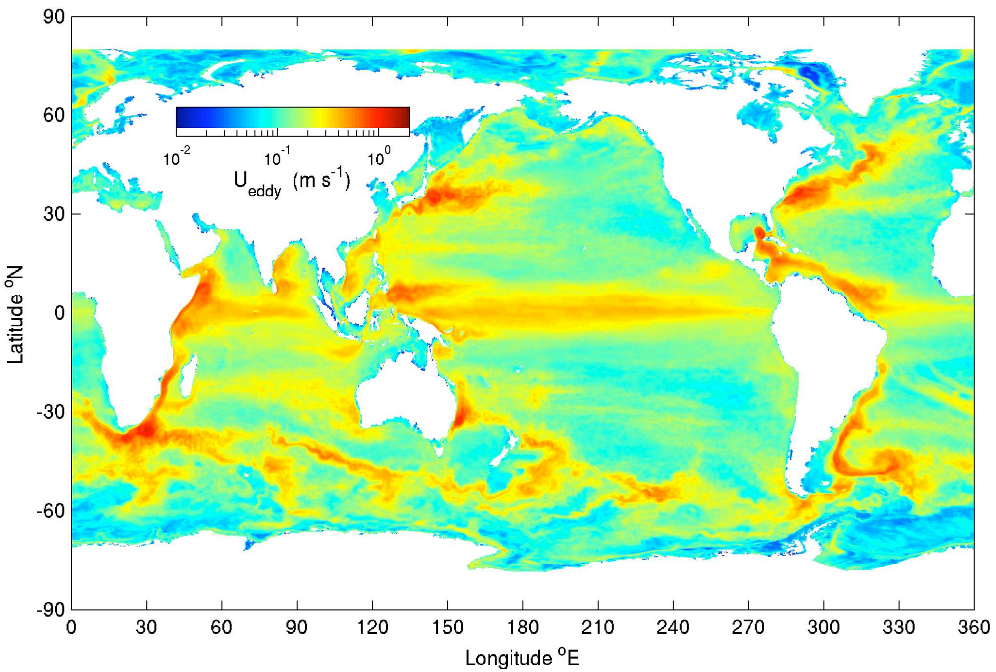


U_{eddy} implied in 1° GFDL coupled model

(assuming local balance between eddy energy production and local decay, 60 days^{-1}):



U_{eddy} diagnosed in 1/8° ECCO2 simulation:



Conclusions

New framework for parameterizing eddy fluxes and diagnosing models:
- down-gradient flux of potential vorticity or isopycnal elevation;
- conserves energy;
- eddy transfer coefficient ~ eddy energy.

Eddy energy is a prognostic variable.

Enforcing the conservation laws
$\Rightarrow$ classical stability conditions emerge naturally as the criteria
controlling the growth and decay of eddy energy

Can mix potential vorticity without creating spurious energy sources.

Preliminary diagnostic results from OGCMs encouraging.

Need to develop much better models for the dispersion and dissipation of eddy energy.